41st Saas-Fee Course
From Planets to Life
3-9 April 2011

Lecture 6--Hydrogen escape, Part 2

Diffusion-limited escape/
The atmospheric hydrogen budget/
Hydrodynamic escape

J. F. Kasting

Diffusion-limited escape

- On Earth, hydrogen escape is limited by diffusion through the homopause
- Escape rate is given by (Walker, 1977[*])

$$\Phi_{esc}(H) \cong b_i \, f_{tot}/H_a$$

where

b_i = binary diffusion parameter for H (or H_2) in air

H_a = atmospheric (pressure) scale height

f_{tot} = total hydrogen mixing ratio in the stratosphere

[*] J.C.G. Walker, *Evolution of the Atmosphere* (1977)

- Numerically

$b_i \cong 1.8\times10^{19}$ cm^{-1}s^{-1} (avg. of H and H_2 in air)

$H_a = kT/mg \cong 6.4\times10^{5}$ cm

so

$\Phi_{esc}(H) \cong 2.5\times10^{13} f_{tot}(H)$ (molecules cm^{-2} s^{-1})

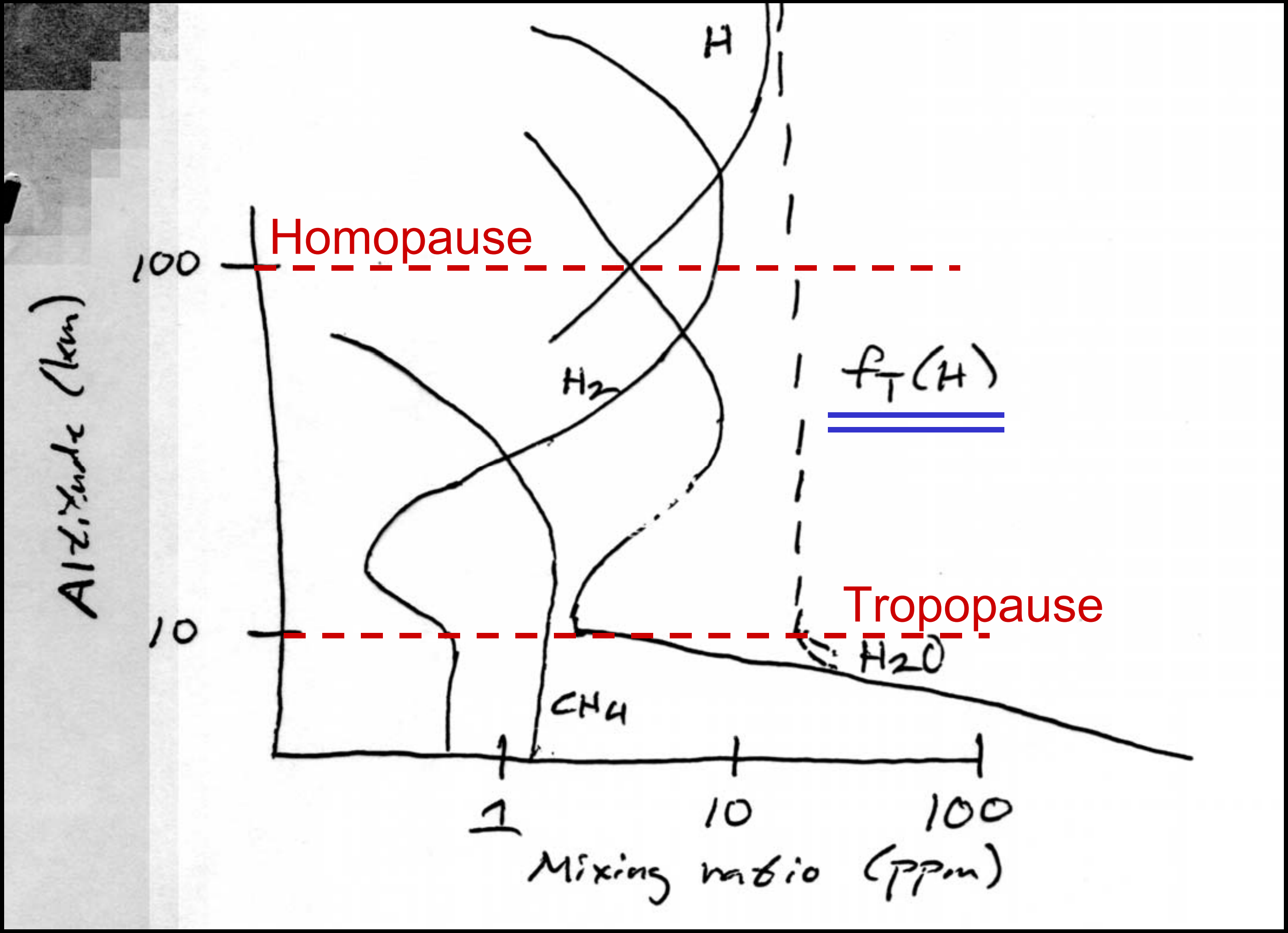
Total hydrogen mixing ratio

- In the stratosphere, hydrogen interconverts between various chemical forms
- Rate of upward diffusion of hydrogen is determined by the *total hydrogen mixing ratio*

$$f_{tot}(H) = f(H) + 2\ f(H_2) + 2\ f(H_2O) + 4\ f(CH_4) + \ldots$$

- $f_{tot}(H)$ is nearly constant from the tropopause up to the homopause (i.e., 10-100 km)

Total hydrogen mixing ratio



Diffusion-limited escape

- Let's put in some numbers. In the lower stratosphere

  $f(H_2O) \cong$ 3-5 ppmv $= (3\text{-}5)\times10^{-6}$

  $f(CH_4) =$ 1.6 ppmv $= 1.6\times10^{-6}$

- Thus

  $$f_{tot}(H) = 2\,(3\times10^{-6}) + 4\,(1.6\times10^{-6})$$
  $$\cong 1.2\times10^{-5}$$

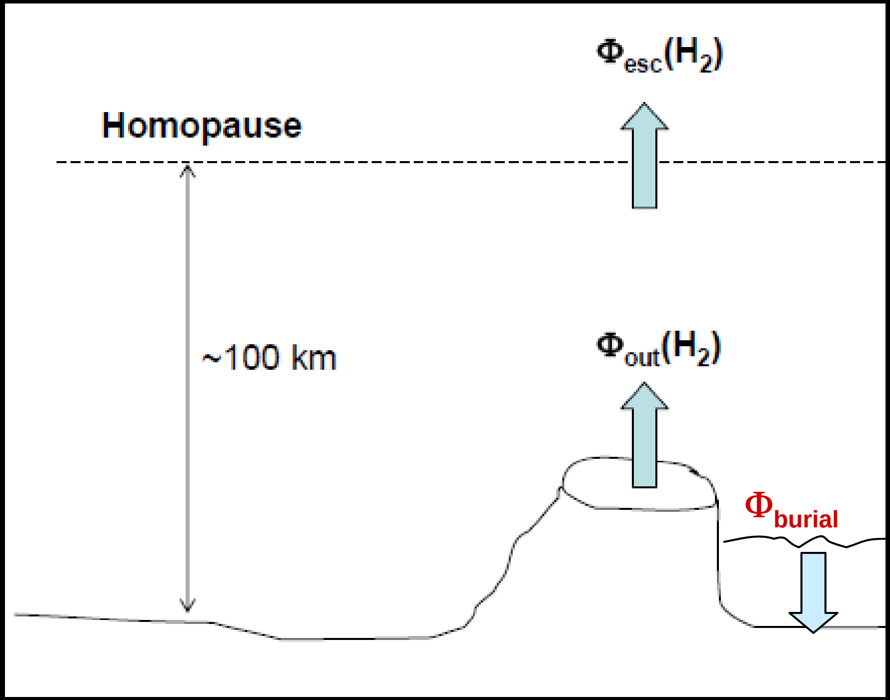
  so the diffusion-limited escape rate is

$$\Phi_{esc}(H) \cong 2.5\times10^{13}\,(1.2\times10^{-5}) = 3\times10^{8}\ \text{cm}^{-2}\ \text{s}^{-1}$$

Hydrogen budget on the early Earth

- For the early earth, we can estimate the atmospheric H_2 mixing ratio by balancing volcanic outgassing of H_2 (and other reduced gases) with the diffusion-limited escape rate
  - Reducing power (available electrons) is also going into burial of organic carbon, but this is slow, at least initially
- Gases such as CO or CH_4 get converted to H_2 via photochemistry

$$CO + H_2O \rightarrow CO_2 + H_2$$
$$CH_4 + 2\,H_2O \rightarrow CO_2 + 4\,H_2$$



$$\Phi_{burial} = \Phi_{burial}(CH_2O)$$
$$CO_2 + 2\,H_2 \rightarrow CH_2O + H_2O$$

Early Earth H budget (cont.)

- Equating loss to space with volcanic outgassing gives (working in units of H_2 this time):

$$\Phi_{esc}(H_2) = \Phi_{out}(H_2) + 2\Phi_{burial}(CH_2O)$$

$$\frac{b_i f_{tot}(H_2)}{H_a} = \Phi_{out}(H_2)$$

but

$$b_i \simeq 1.8\times10^{19}\ \text{cm}^{-1}\text{s}^{-1}$$

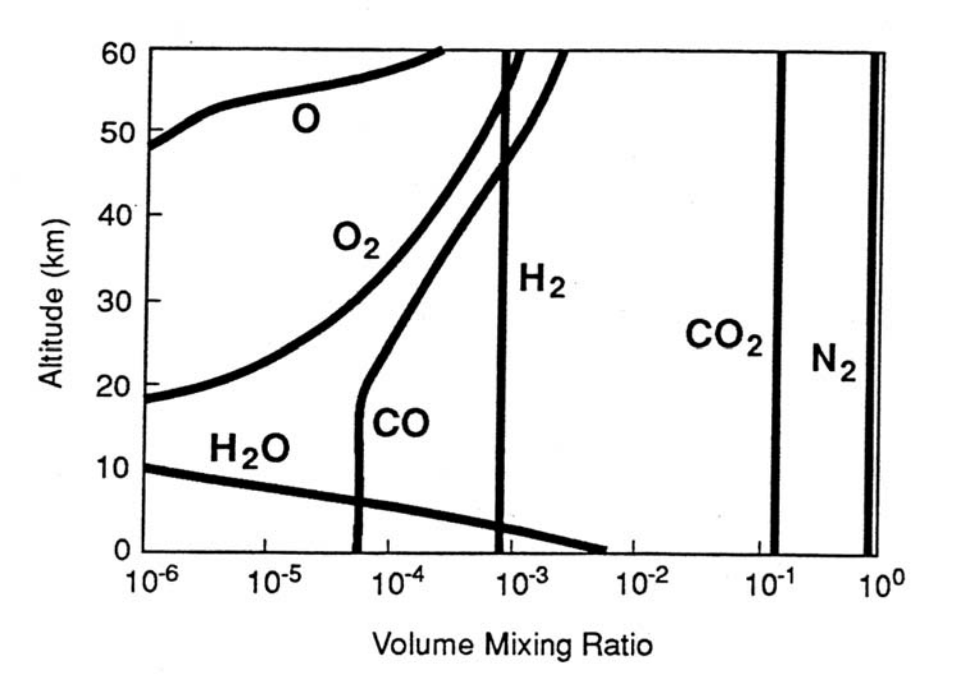
$$H_a \simeq 6.4\times10^{6}\text{cm}$$

so, get

$$2.5\times10^{13} f_{tot}(H_2)\ \text{cm}^{-1}\text{s}^{-1} \simeq \Phi_{out}(H_2)$$

- For a typical (modern) H_2 outgassing rate of 1×10^{10} cm^{-2}s^{-1}, get $f_{tot}(H_2) \cong 4\times10^{-4}$ (mostly as H_2 and CH_4)
- This could be significantly higher on the early Earth
- Modern H_2 outgassing rate estimated by ratioing to outgassing of H_2O and CO_2

Weakly reduced atmosphere



- So, this is how we derive the basic chemical structure of a weakly reduced atmosphere
- H_2 concentrations in the prebiotic atmosphere could have been *higher* than this if volcanic outgassing rates were higher or if H escaped more slowly than the diffusion-limited rate, but they should not have been lower
- Consequently, this gives us an *upper limit on prebiotic O_2*

J. F. Kasting, *Science* (1993)

Hydrogen escape: summary

- Hydrogen escapes from terrestrial planets by a variety of *thermal* and *nonthermal* mechanisms
  - Thermal mechanisms include *both* Jeans escape and hydrodynamic escape
- H escape can be limited either at the *homopause* (by diffusion) or at the *exobase* (by energy)
- For the early Earth, assuming that H escape was diffusion-limited, and using modern H_2 outgassing rates, provides a lower bound on the atmospheric (H_2+ 2 CH_4) mixing ratio and an upper bound on pO_2
- Hydrogen can drag off heavier elements as it escapes, provided that the escape flux is fast enough

Hydrodynamic escape

(We’ll only do this if time allows on Thursday)

- What happens, though, if the atmosphere becomes very hydrogen-rich?
- It is easy to show that the assumptions made in all of the previous analyses of hydrogen escape break down…

Breakdown of the barometric law

- Normal barometric law
- As $z \to \infty$, p goes to zero, as expected

$$dp = -\rho g dz$$

$$p = nkT = \rho kT / m$$

$$so$$

$$\rho = mp / kT$$

$$\frac{dp}{p} = -\frac{mgdz}{kT} \equiv -\frac{dz}{H_a}$$

Integrate:

$$\ln p = -\frac{z}{H_a} + const.$$

$$p = p_0 \exp\left(-\frac{z - z_0}{H_a}\right)$$

Breakdown of the barometric law

- Now, allow *g* to vary with height
- As $r \to \infty$, *p* goes to a constant value
- This suggests that the atmosphere has infinite mass!
- How does one get out of this conundrum?

$$g = \frac{GM}{r^2}$$

$$\frac{dp}{p} = -\frac{GMm\,dr}{r^2 kT}$$

Integration yields:

$$\ln p = \frac{GMm}{rkT}$$

$$p = p_0 \exp\left(\frac{GMm}{kT}\left(\frac{1}{r} - \frac{1}{r_0}\right)\right)$$

Answer(s):

Either

- The atmosphere becomes *collisionless* at some height, so that pressure is not defined in the normal manner
  - This is what happens in today's atmosphere

or

- The atmosphere is *not hydrostatic*, *i.e.*, it must expand into space

Fluid dynamical equations

(1-D, spherical coordinates)

Conservation of mass

$$\frac{\partial \rho}{\partial t} = -\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \rho u\right) \tag{B4.1}$$

Conservation of momentum

$$\frac{\partial(\rho u)}{\partial t} = -\rho u \frac{\partial u}{\partial r} - \frac{\partial p}{\partial r} - \rho \frac{GM}{r^2} \tag{B4.2}$$

Conservation of energy

$$\rho c_p \frac{\partial T}{\partial t} = -\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \kappa \frac{\partial T}{\partial r}\right) - \frac{kTu}{m}\frac{\partial \rho}{\partial r} + \rho c_v u \frac{\partial T}{\partial r} + q \tag{B4.3}$$

Bernoulli's equation

- If the energy equation is ignored, and we take the solution to be isothermal (T = const.) and time-independent, then the mass and momentum equations can be combined to yield *Bernoulli's equation*

$$\frac{1}{u}\frac{du}{dr}\left(1-\frac{u^2}{u_0^2}\right)=\frac{2r_0}{r^2}-\frac{2}{r}$$

$$u_0^2=\frac{kT}{m} \qquad r_0=\frac{GMm}{2kT}$$

- This equation can be integrated to give

$$\ell n\frac{u}{u_0}-\frac{1}{2}\left(\frac{u}{u_0}\right)^2=-\frac{2r_0}{r}-2\ell n\frac{r}{r_0}+const. \qquad \text{(B4.12)}$$

Transonic solution

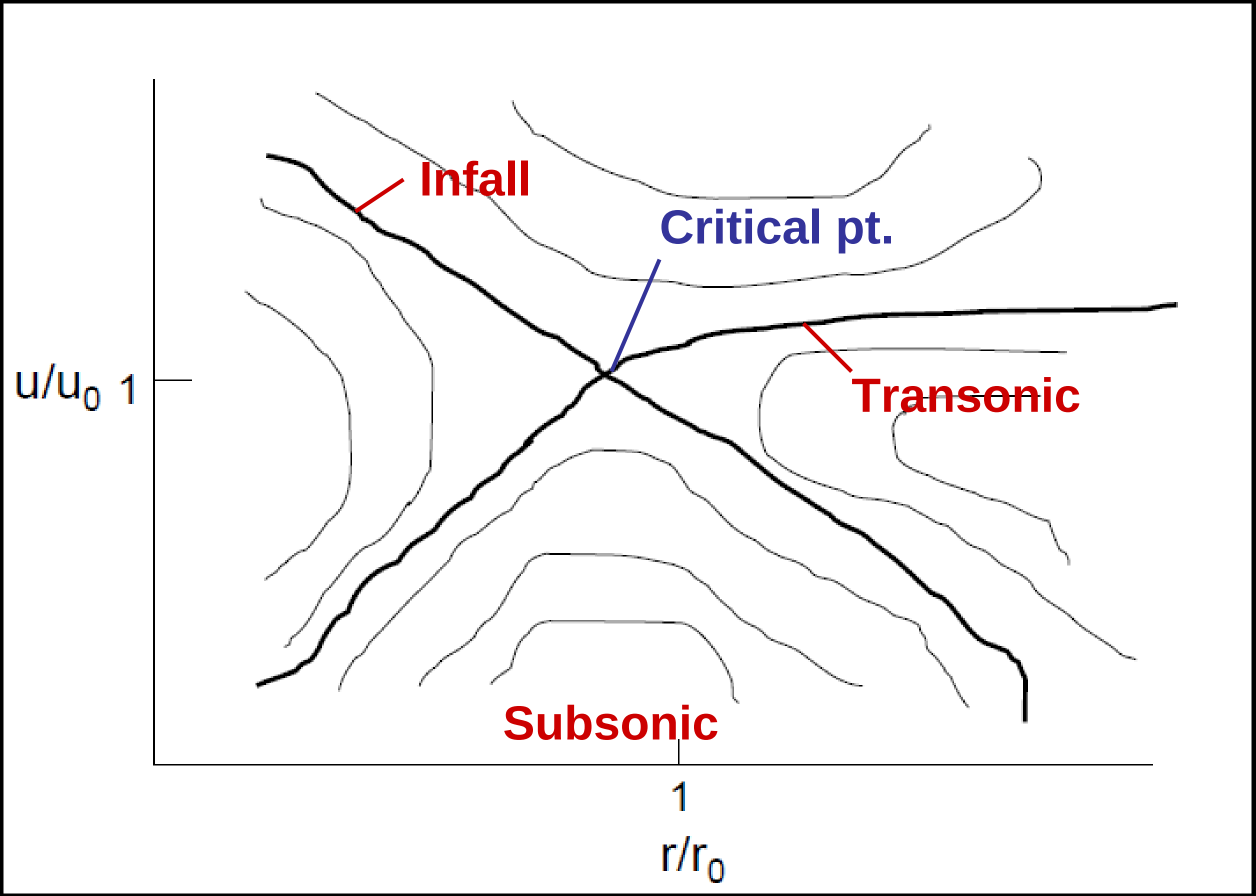
- Bernoulli's equation give rise to a whole family of mathematical solutions
- One of these is the *transonic solution*

$$\ell n \frac{u}{u_0} - \frac{1}{2}\left(\frac{u}{u_0}\right)^2 + \frac{2r_0}{r} + 2\ell n \frac{r}{r_0} - \frac{3}{2} = 0 \qquad \text{(B4.13)}$$

- This solution goes through the *critical point* (r_0, u_0), where both sides of the differential form of the equation vanish

- (Draw solutions to Bernoulli's equation on board)

Solutions to Bernoulli's equation



- The solutions of physical interest are the *transonic* solution, the *infall* solution, and the *subsonic* solutions

Mass fractionation during hydrodynamic escape

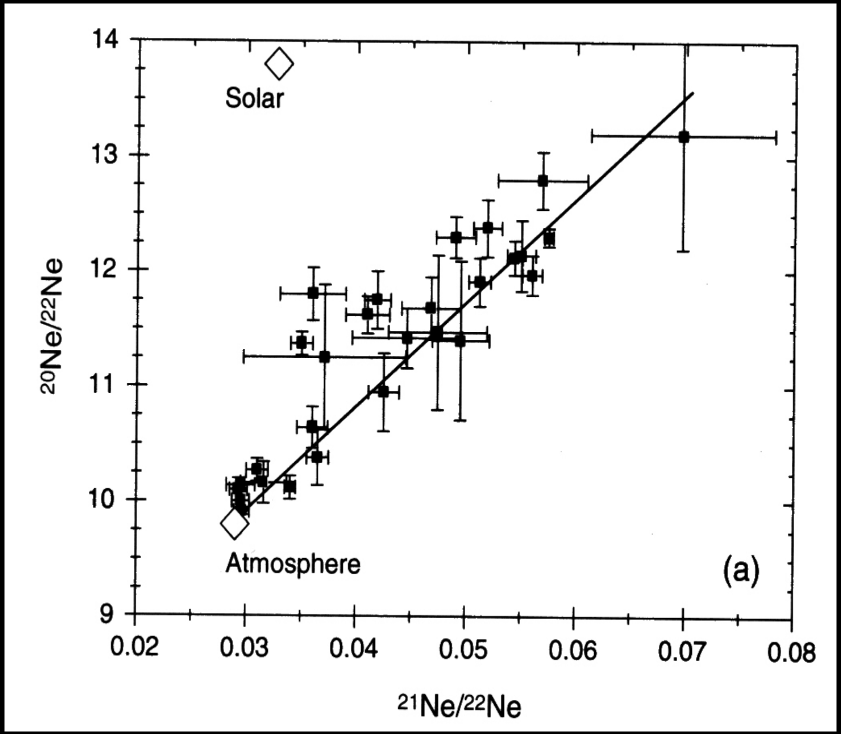
- Hydrodynamic escape of hydrogen can fractionate elements and isotopes by carrying off heavier gases
- This becomes important for gases lighter than the *crossover mass*

$$m_c = m_1 + \frac{kTF_1}{bgX_1}$$

m_1 = mass of hydrogen atom (or molecule)
F_1 = escape flux of hydrogen
X_1 = mixing ratio of hydrogen
b = binary diffusion coefficient (= D_i/n)

Neon isotopes

- 3-isotope plots can be used to distinguish gases coming from different sources
- Data shown are neon isotope ratios in MORBs (midocean ridge basalts)
- Earth's atmosphere is depleted in ^{20}Ne relative to ^{22}Ne
  - ^{21}Ne is radiogenic and is simply used to indicate a mantle origin
- Mantle Ne resembles solar Ne
  - Ne is thought to have been incorporated by solar wind implantation onto dust grains in the solar nebula
- The atmospheric $^{20}Ne/^{22}Ne$ ratio can be explained by rapid hydrodynamic escape of hydrogen, which preferentially removed the lighter Ne isotope



Ref: Porcelli and Pepin, in R. M. Canup and K. Righter, eds., *Origin of the Earth and Moon* (2000), p. 439

Energy-limited escape

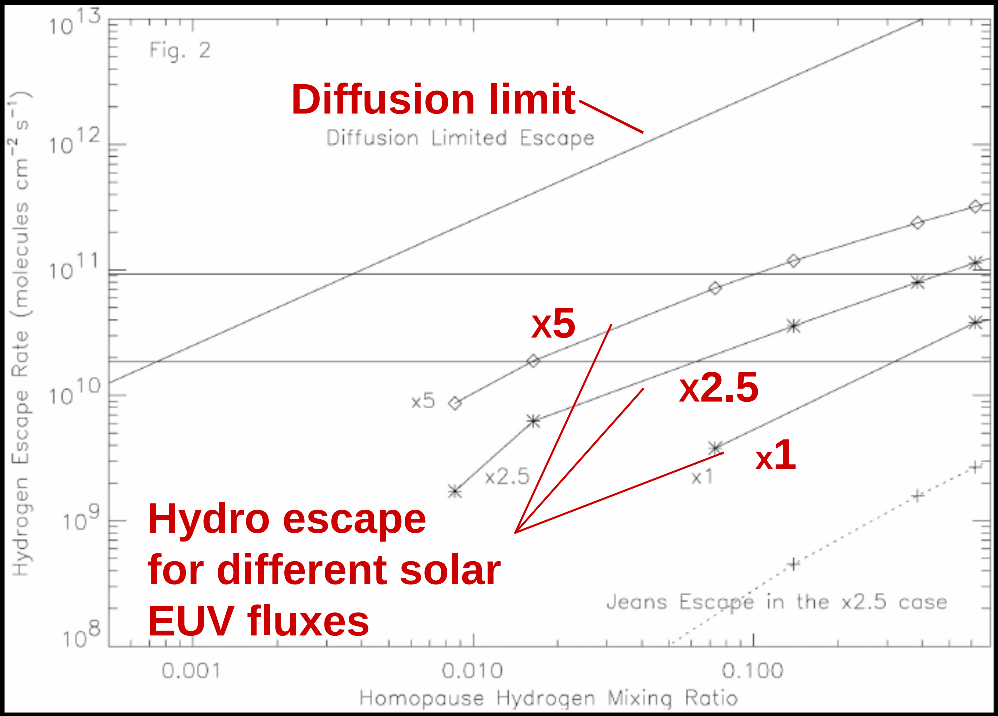
- The energy needed to power hydrodynamic escape is provided by absorption of solar EUV radiation ($\lambda < 900$ nm)
  - The solar flux at these wavelengths is ~1 erg/cm^2/s
- The *energy-limited escape rate*, Φ_{EL} is given by

$$\frac{GMm}{r}\Phi_{EL} = \varepsilon S$$

S = solar EUV flux
ε = EUV heating efficiency (typically 0.15-0.3)

How fast is hydrodynamic escape?

- Preliminary results (for a pure H_2 atmosphere) suggest that hydrodynamic escape will be *slower* than diffusion-limited escape
- This conclusion needs to be verified with a model that includes realistic upper atmosphere composition, chemistry, and physics
  - This is a good project for mathematically inclined students



F. Tian et al., Science (2005)