On the PDS of GRB light curves

Mimoza Hafizi, Sonila Boçi, Tirana University, Albania

Astrogroup, Tirana University

Our focus:

- Temporal properties of GRBs and their relations to the internal properties:
  - lag-luminosity relation
  - duration-luminosity relation
  - lags as a consequence of the GRBs spectral evolution
- Amati-like relations linking the energy with the spectral properties;
- Internal shock processes in the relativistic wind;
- Opening angles in beamed GRBs.

Light curves versus PDS

The light curves of GRBs, $c(t)$:

Many random peaks;

Only few percent of them exhibit a single pulse structure;

Diverse and composite structure which appears to be the result of a complex distribution of several pulses;

The total duration is the first timescale used to characterize them;

The typical pulse duration seems to be relevant as a second timescale.

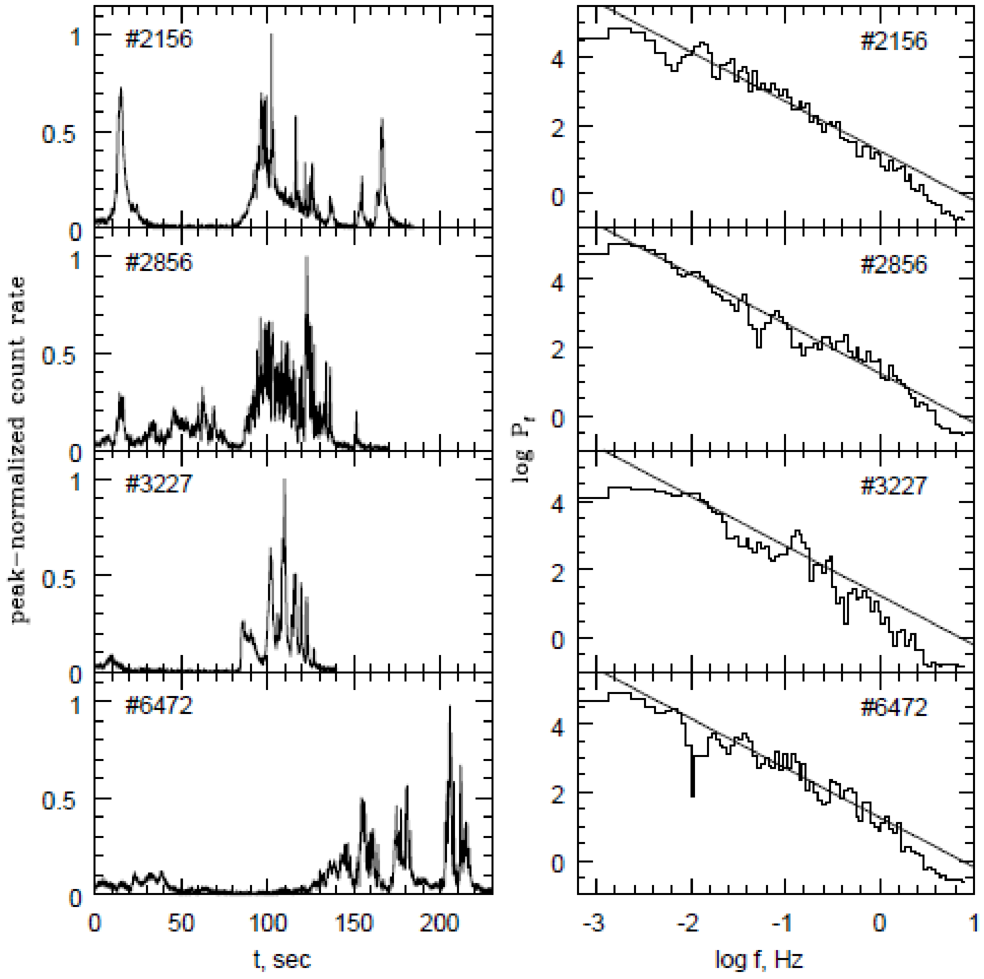
The Fourier domain of frequencies, PDS,

$c(f) = \int_{-\infty}^{+\infty} dt c(t) e^{2\pi i f t}$ and the power density

$P_f = C_f C_f^*$ ($C_k = \sum_{m=0}^{N-1} c_m e^{2\pi i m k/N}$, for N discrete data DFT, $P_k = |C_k|^2$):

Simplier behaviour: power-law.

Particular interest on long bursts-spectral analysis over e large range of time.



Beloborodov, A. M., Stern, B. E., & Svensson, R. 2000, ApJ

Computing PDS

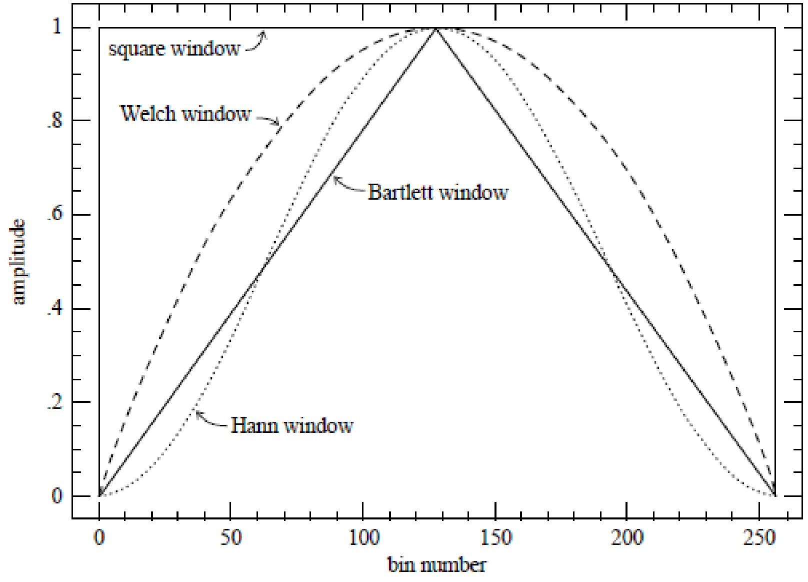
One estimator of PDS, called **periodogram**, is given by:

$$P(f_k)=\frac{1}{N^2}\left[|C_k^2|^2+|C_{N-k}^2|^2\right], k=1,2,\dots,(\frac{N}{2}-1);$$

$P(f_k)$ is considered as the average of $P(f)$ over a narrow window function,

$$W(f)=\frac{1}{N^2}\left[\frac{sin\pi f}{sin\pi f/N}\right]^2,$$

the Fourier transform of the **rectangular function**.



The PDS and DFT for N discrete data leads to $O(N^2)$ arithmetic operations.

There is an algorithm FFT, which helps to find the same result more quickly, with $Nlog(N)$ operations. The difference in speed is enormous, especially for long data series.

N used is an integer power of 2.

$W(f)$ is not zero outside the corresponding frequency interval ⇒ periodogram estimate is influenced from other frequencies outside the interval, “leaks from one frequency to another”.

A mean of the leakage correction is the **data windowing**. Instead of the rectangular function, one choses a window function that changes more gradually from zero to its maximum and then back to zero.

In our calculations we use **Bartlett window**, but there are several different ways.

The variance

The variance σ^2 of the periodogram estimate is independent on N ***(William H. Press, Saul A. Teukolsky, William T. Vetterling, Brian P. Flannery , "Numerical recipes in FORTRAN" 1997).***

We can produce estimates at a greater number of discrete frequencies, but with the same high standard deviation, 100%: $\sigma \propto P_k$.

There are techniques for reducing the variance of the estimates:

- Technique of finer data (we use in Tirana):
  - Have a higher number of data in time series, KN instead of N ;
  - Partition the original KN data into N segments each of K consecutive sampled points;
  - Fourier transform of each sequence, one point from each segment; to produce periodogram estimates;
  - Average the K periodogram estimates at each frequency;
  - The standard deviation is reduced : $\sigma \propto \frac{1}{\sqrt{K}}$.
- Technique of Montecarlo simulation of synthetic GRB ***(Ukwatta T., K. S. Dhuga, D. C. Morris, G. MacLachlan, W. C. Parke, L. C. Maximon, A. Eskandarian, N. Gehrels, J. P. Norris, and A. Shenoy., 2011, MNRAS)***
  - Have a real burst $c(t)$;
  - simulate K light curves around the real one, based on the real one and some fluctuations around by a montecarlo random number η generated from a Gaussian distribution, with mean value equal to zero and standard deviation equal to one:
    $$c_{bin}^{simul} = c_{bin}^{real} + \eta c_{bin}^{real-error}$$
  - Find the individual PDS of each one and average. The standard deviation is reduced: $\sigma \propto \frac{1}{\sqrt{K}}$.
- Leahy normalization ***(Guidorzi C., 2011, MNRAS)***
  - $P(f_k) = \frac{1}{N}\sum_{m,l} c_m\, c_l e^{2\pi i(m-l)k/N}$;
  - The variance is $\sigma \propto \sqrt{P_k}$.

Average PDS

Averaging over a sample of long GRBS is an way to extract un underlying law from the noisy individual PDSs.

It is assumed that time series due to different GRBs are many realizations of the same stochastic process.

Remark: the wide variety of light curves exhibited by GRBs is potentially indicative of different stochastic, emission and scattering processes.

- We sum up the PDS of individual bursts (after some normalization) and divide the result by the number N of bursts.
- The standard distribution of the individual PDS around $\overline{P_f}$ follows $\frac{\Delta P_f}{\overline{P_f}} \propto N^{-1/2}$.
  - The light curves are normalized to their peak (needed for increasing the weight of relatively weak bursts in the sample)(***Beloborodov et al. 2000, ApJ; Guidorzi, C., Margutti, R., Amati, L., et al. 2012, MNRAS***).
  - Or the averaging is performed inside a sole group of variability, taking into account also a kind of pseudoredshift (obtained through empirical relations) (***Lazzati, D. 2002, MNRAS***).
  - Or the averaging is performed inside subclasses of GRBs found based on the autocorrelation function (***Borgonovo, L., Frontera, F., Guidorzi, C., Montanari, E., Vetere, L., & Soffitta, P. 2007***) and considering the measured redshift.

Individual PDS

The average PDS provides no clues on the variety of properties of individual GRBs.

The key point of studying individual versus averaged PDS is that one can investigate the possible connection between PDS and their key properties of prompt emission.

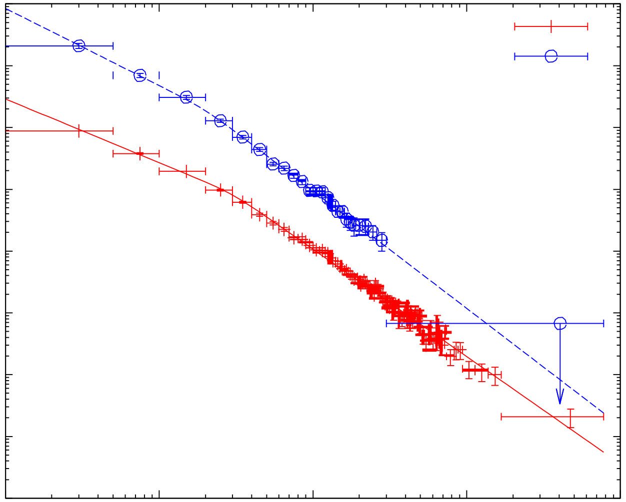
- **Simulating around the same GRB (*Ukwatta T. et al., 2011*)**
  - Find the individual PDS of an individual light curve;
  - Simulate 100 light curves around the real one, based on the real one and some fluctuations around by a montecarlo random number generated from a Gaussian distribution;
  - Find the standard deviation of 100 simulated PDS and calculate the individual PDS with this uncertainty.

- **A Leahy formula for uncertainty at each frequency (*C. Guidorzi, S. Dichiara and L. Amati, 2016; S. Dichiara, C. Guidorzi, L. Amati, F. Frontera and R. Margutti, 2016*)**
  - Each GRB time profile is considered individually as the unique sample of a unique stochastic process, which is different from other GRBs.
  - PDSs are calculated assuming Leahy normalization.
  - A Leahy formula for calculating the correct uncertainty of the PDS at each frequency, as a function of PDS itself.

Modelling the PDS-Averaged PDS

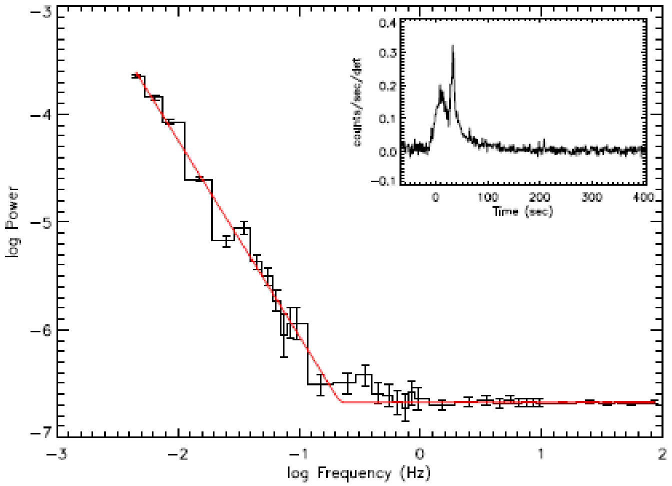
As found in several independent data sets:

- The average PDS of long GRBs is described by a power law extending over two frequency decades, from about 10^{-2} to 1 or 2 Hz.
- The power-law index lies in the range 1.5-2.
- There is evidence for a break around 1-2 Hz for the harder (>~100 keV) energy channels.



(*Beloborodov et al. 2000*; ***Ryde et al. 2003***; *Guidorzi et al. 2012*; ***Dichiara, S., Guidorzi, C., Amati, L., & Frontera, F. 2013a, MNRAS***)- figure top.

A threshold noise crossing frequency f_{th} is evident-figure down (***Ukwatta et al. 2011***)- figure bottom.



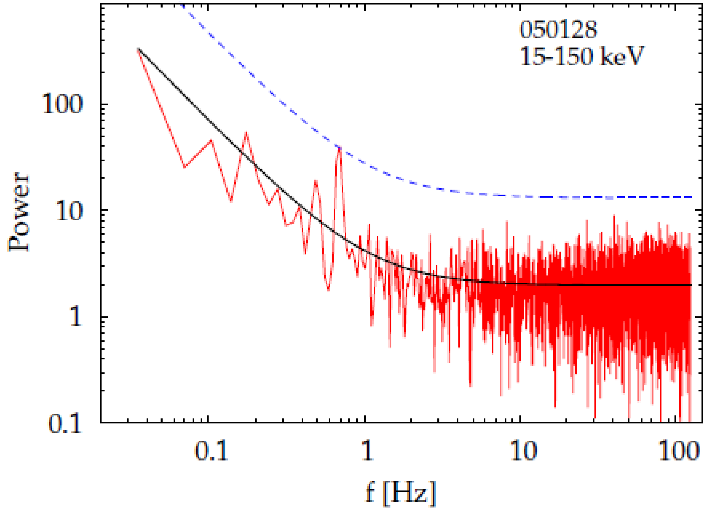
Modelling the PDS-Individual PDS

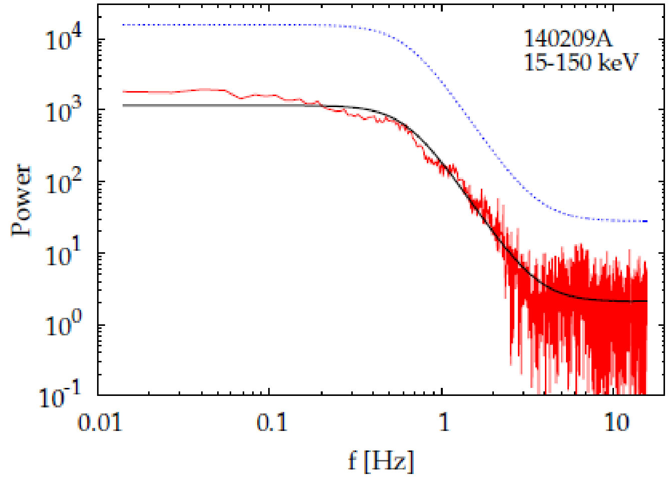
Because of the limited duration and of the statistical properties involved, modeling the PDS of individual GRBS is challenging.

- Several models are used.
- The simpler one is a mere power-law: $S_{PL} = Af^{-\alpha}$+B, B is the white noise constant (top).
- Otherwise, the broken power law model introduces the break frequency $S_{BPL} = N\left[1+\left(\frac{f}{f_b}\right)^{\alpha}\right]^{-1} + B$ (bottom).

For individual PDS, the power law index lies in the range 1.5-4 with some exceptions over 6.

Most of the break frequency values correspond to timescales that are around one second.



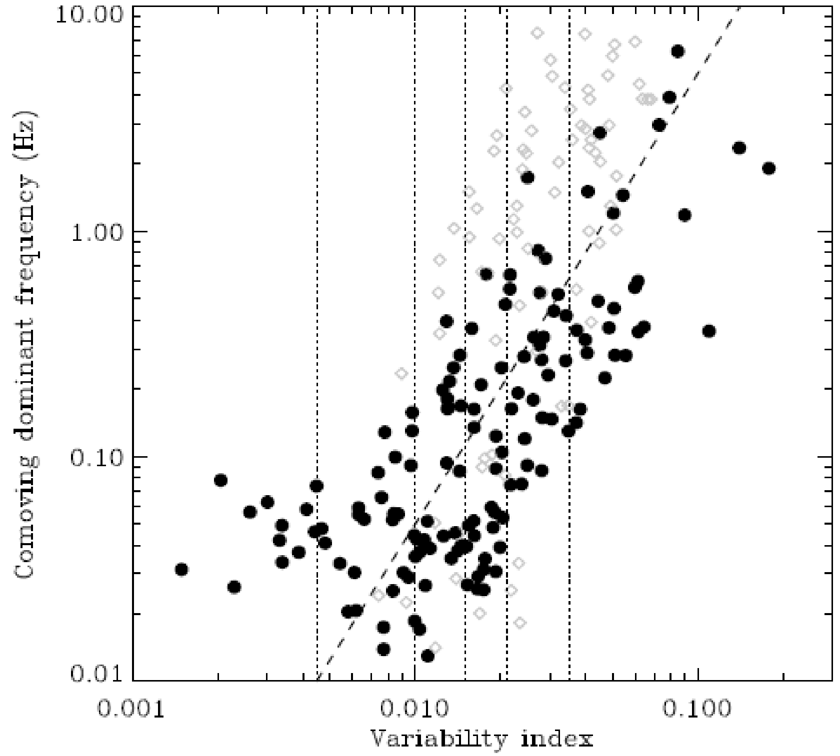


*C. Guidorzi, S. Dichiara and L. Amati, **2016***

Relations PDS-Variability

The dominant frequency-variability measure

Lazzati, D. 2002, MNRAS

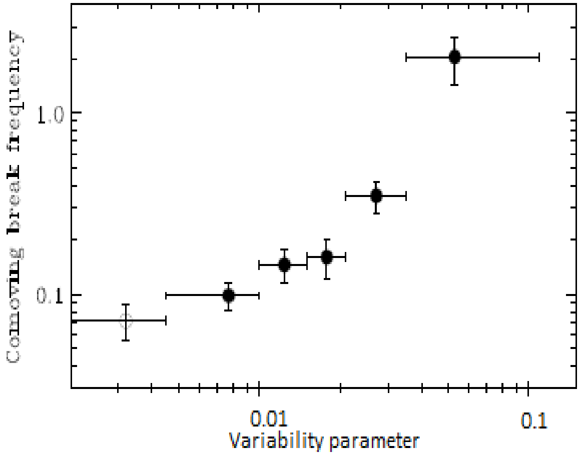


Variability measure defined by Fenimore&Ramirez-Ruiz 2000

(average mean-square of the variations).

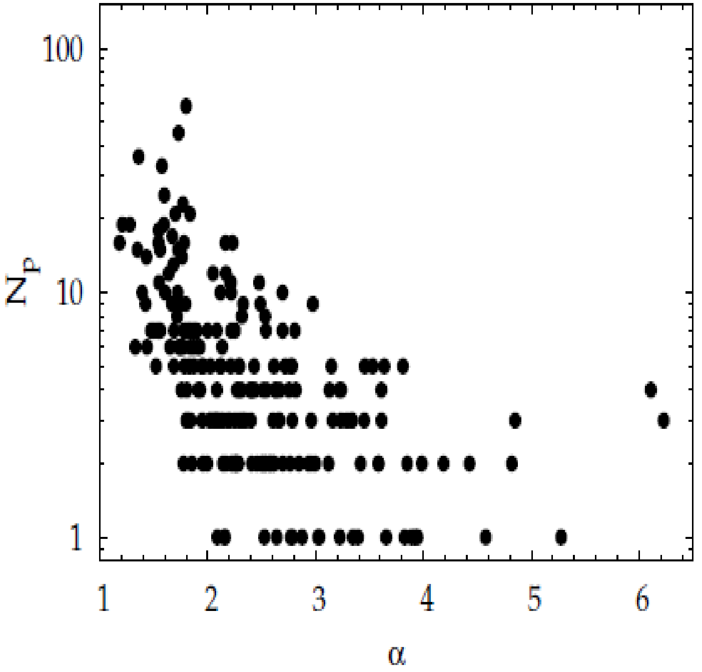
The break frequency-variability measure

Lazzati, D. 2002, MNRAS



Number of pulses seem to correlate with the slope α

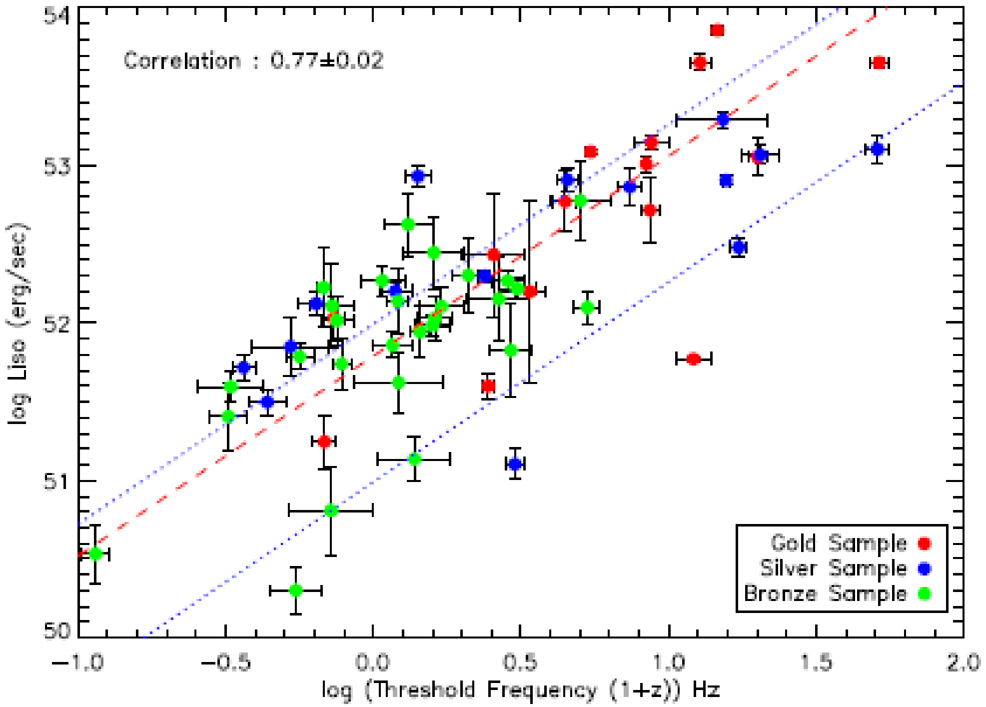
C. Guidorzi, S. Dichiara and L. Amati, 2016;



Relations PDS-GRB energy

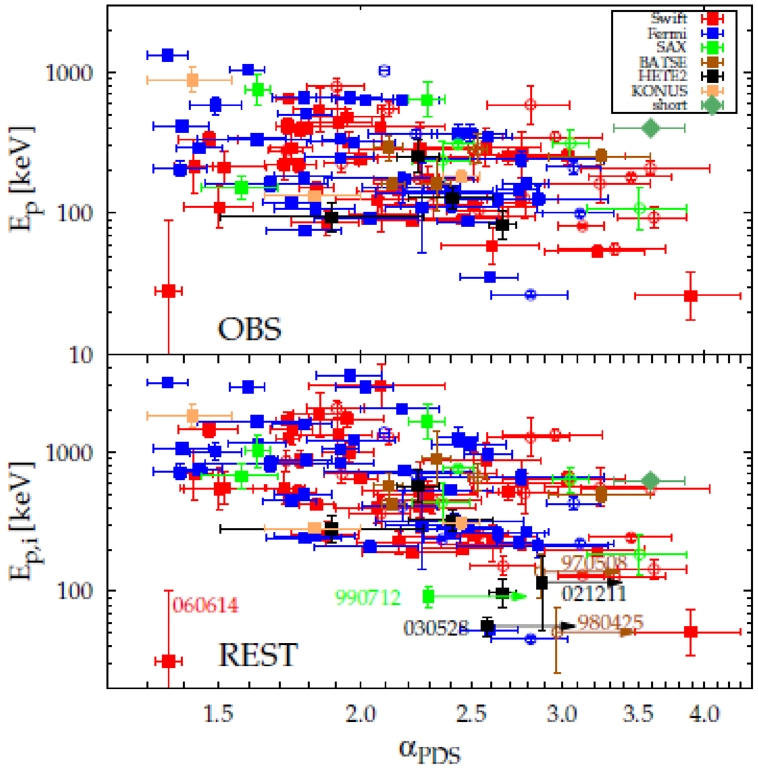
Threshold frequency-isotopic peak luminosity.
The correlation coefficient is 0.57 ± 0.03.

Ukwatta et al. 2011



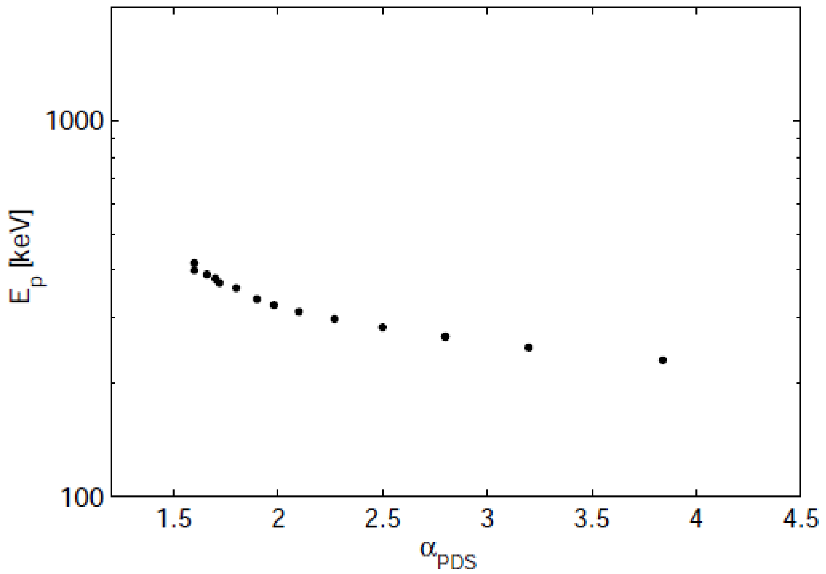
Correlation E_{peak}-slope.

Dichiara et al. 2013



Relation E_{peak}- slope: synthetic pulses

- Taking different peak luminosities in the source, we draw synthetic pulses:
  - With duration based on luminosity-duration relation
    $L = 3.4x10^{52} t_p^{-0.85}$ (Hakkila et al., 2008)
  - With $c(t)$ following
    $$c(t) = F_m \left(\frac{t}{t_m}\right)^r \left(\frac{d}{d+r} + \frac{r}{d+r}\left(\frac{t}{t_m}\right)^{r+1}\right)^{-\frac{r+d}{r+1}}$$ (Kocevski et al., 2003)
    $d, r = 2.4, 1.5,\ t_m = 0.323 t_p (1+z)^{0.6}$ (Kocevski et al., 2003),
    F_m peak photon flux found for $z = 1$
  - With peak energy $E_p = 380 \left(\frac{L}{1.6x10^{52}}\right)^{0.43}$ (Ghirlanda et al., 2005)
- We calculate PDS of the pulse and find the slope inside the interval $(0.3 - 1)$ Hz.
- We draw the relation Epeak-slope.



Conclusions

Is relation E_{peak} - slope another confirmation of correlations between temporal and spectral properties?