Formation of planets by gravitational instability

Wilhelm Kley Institut für Astronomie & Astrophysik & Kepler Center for Astro and Particle Physics Tübingen





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Lecture overview:

- 7.1 Introduction
- 7.2 Theory
- 7.3 Disk fragmentation
- 7.4 Planet evolution



(Credit: Farzana Meru)

7.1 Introduction: The system HR 8799



Star: HR 8799

Constellation: Pegasus Brightness: 6th mag Mass: $1.5M_{\odot}$ Age: 60 mio. yrs Distance: 130 lj.

3 Planets (2008) 4th Planet (2010)

Note: Star very close to the first confirmed exoplanetary system 51 Peg

7.1 Introduction: The system HR 8799



Star: HR8799 Constellation: Pegasus Keck-Telescope (Hawaii) 2 epochs

3 Planets (2008):

Distance: 24, 38, 68 AU Masses: 10, 10, 7 M_{Jup}

Planet motion: for the first time direct observed!

4th planet (2010) 14.5 AU

More systems:

- Fomalhaut
- β Pictoris

7.1 Introduction: The system HR 8799

The planetary system HR 8799 in comparison to the Solar System The *x*-axis is compressed by $\sqrt{L_{HR8799}/L_{\odot}}$ with $L_{HR8799} = 4.9L_{\odot}$. I.e. the planets are about twice as far away but have the same equilibrium temperatures as the Solar planets (Marois ea., Nature, 2010)



7.1 Introduction: Core formation time scale

How did the system HR 8799 form ?

Analyze growth time scale for planet at Neptune's present location Using the result from Lecture 2 (Sect. 2.1)

$$\frac{dm_{\rho}}{dt} = \frac{\sqrt{3}}{2} \Sigma_{\text{part}} \Omega_{\kappa} \pi R_{\rho}^{2} \left[1 + \left(\frac{v_{\text{esc}}}{v_{\text{rel}}} \right)^{2} \right]$$
(1)

we obtain with $\Sigma_{part}=1~g/cm^3$

$$\tau_{grow} = \frac{m_{\rho}}{dm_{\rho}/dt} \approx 5 \cdot 10^{10} F_{grav}^{-1} \quad \text{yr}$$
 (2)

Unless the gravitational focussing factor is very large ($\sim 10^4)$ this time scale is very long indeed.

In Solar System Neptune has migrated outward \rightarrow problem reduced But need alternative scenario for distant, directly imaged planets

7.1 Introduction: Study stability of disks

To analyse the stability properties of disks, basically two methods can be applied

- Linear Stability analyses study the evolution of small perturbances in the linearised equations derive dispersion analyses for sinusoidal pertubations and look for stability criterion
- Non-linear evolution of disks numerical simulations of self-gravitating disks study the onset of instability and subsequent evolution compare to linear results perform parameter studies

Study both in following sections

7. Planets formed by self-gravity: Organisation

Lecture overview:

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$$\frac{\partial u_r}{\partial t} + u_r \frac{\partial u_r}{\partial r} + \frac{v_{\phi}}{r} \frac{\partial u_r}{\partial \phi} - \frac{v_{\phi}^2}{r} = -\frac{\partial}{\partial r} \left(h + \Psi + \Psi_*\right), \quad (1)$$

$$\frac{\partial v_{\phi}}{\partial t} + u_r \frac{\partial v_{\phi}}{\partial r} + \frac{v_{\phi}}{r} \frac{\partial v_{\phi}}{\partial \phi} + \frac{v_{\phi} u_r}{r} = -\frac{1}{r} \frac{\partial}{\partial \phi} \left(h + \Psi + \Psi_*\right), \quad (2)$$

$$\frac{\partial \sigma}{\partial t} + \frac{1}{r} \frac{\partial}{\partial r} (r \sigma u_r) + \frac{1}{r} \frac{\partial}{\partial \phi} (\sigma v_{\phi}) = 0 , \qquad (3)$$

$$\Psi(r, \phi) = -G \int_{R_{\rm in}}^{R_D} \sigma(r') r' dr' \\ \times \int_0^{2\pi} \frac{d\phi'}{\sqrt{r^2 + r'^2 - 2rr' \cos \phi' + \eta^2(r)}} \,. \tag{4}$$

(Laughlin ea 1999)

7.2 Theory: 2D-Equations of motion (flat disk)

Conservation of mass

$$\frac{\partial \Sigma}{\partial t} + \nabla \cdot (\Sigma \mathbf{u}) = 0 \qquad \mathbf{u} = (u_r, u_{\varphi}) = (u, r\omega)$$
(3)

Radial Momentum

$$\frac{\partial(\Sigma v)}{\partial t} + \nabla \cdot (\Sigma v \mathbf{u}) = \Sigma r \omega^2 - \frac{\partial P}{\partial r} - \Sigma \frac{\partial \psi}{\partial r}$$
(4)

Angular Momentum

$$\frac{\partial(\Sigma r^2\omega)}{\partial t} + \nabla \cdot (\Sigma r^2 \omega \mathbf{u}) = -\frac{\partial P}{\partial \varphi} - \Sigma \frac{\partial \psi}{\partial \varphi}$$
(5)

Equation of state

$$\boldsymbol{P} = \boldsymbol{P}(\boldsymbol{\Sigma}) \tag{6}$$

Gravitational potential, star, planet, disk (possibly indirect terms)

$$\psi = \psi_* + \psi_p + \psi_d \tag{7}$$

7.2 Theory: Basic state & Perturbation

Basic state:

Stationary, axisymmetric equilibrium (from radial equation)

$$r\Omega_0^2 - \frac{1}{\Sigma_0} \frac{\partial P_0}{\partial r} - \frac{\partial \psi_0}{\partial r} = 0$$
(8)

• Perturb around basic state:

Consider small pertubations: $f(r, \varphi, t) = f_0(r) + f_1(r, \varphi, t)$

$$\Omega = \Omega_0 + \Omega_1, \quad u = u_0 + u_1, \quad \Sigma = \Sigma_0 + \Sigma_1, \quad P = P_0 + P_1, \quad \psi = \psi_0 + \psi_1$$

- Assumptions:
 - i) perturbations are small: f₁ << f₀ (neglect quadratic terms)
 - *ii*) 2D: Perturbations only in z = 0 plane: $f_1 = f_1(r, \varphi, t)$

7.2 Theory: Linearised equations

• Substitute ansatz into full non-linear equations, drop all higher order terms:

$$\frac{\partial \Sigma_1}{\partial t} + \Omega_0 \frac{\partial \Sigma_1}{\partial \varphi} + \Sigma_0 \frac{\partial u_1}{\partial r} + \Sigma_0 \frac{\partial \Omega_1}{\partial \varphi} = 0$$
(9)

$$\frac{\partial u_1}{\partial t} + \Omega_0 \frac{\partial u_1}{\partial \varphi} - 2r\Omega_0 \Omega_1 = -\frac{c_{s_0}^2}{\Sigma_0} \frac{\partial \Sigma_1}{\partial r} - \frac{\partial \psi_1}{\partial r}$$
(10)

$$\frac{\partial\Omega_1}{\partial t} + \Omega_0 \frac{\partial\Omega_1}{\partial\varphi} + \frac{u_1}{r} \frac{\kappa_0^2}{2\Omega_0} = -\frac{1}{r^2} \frac{c_{s_0}^2}{\Sigma_0} \frac{\partial\Sigma_1}{\partial\varphi} - \frac{1}{r^2} \frac{\partial\psi_1}{\partial\varphi} \quad (11)$$

with *epicyclic frequency*

$$\kappa_0^2 \equiv \frac{1}{r^3} \frac{\partial}{\partial r} \left[(r^2 \Omega_0)^2 \right] = 4\Omega_0^2 + 2\Omega_0 r \frac{\partial \Omega_0}{\partial r}$$

and sound speed

$$c_{s_0} = \left(rac{dP}{d\Sigma}
ight)_0^{1/2}$$

The equations are linear in the perturbed quantities $f_1(r, \varphi, t)$

7.2 Theory: Fourier expansion

Equations: (9) to (11) could be integrated numerically, often better for stability analysis to expand the perturbations into Fourier-series Basic state has no time and azimuthal dependence \Rightarrow Consider perturbations with:

$$f_1 = \tilde{f}_1(r)e^{i(m\varphi - \sigma t)}$$
(12)

then

$$\frac{\partial}{\partial t} \Rightarrow -i\sigma$$
 and $\frac{\partial}{\partial \varphi} \Rightarrow im$ (13)

$$\tilde{\Sigma}_{1}(\sigma - m\Omega_{0}) = -i\Sigma_{0}\tilde{u}_{1}' + \Sigma_{0}m\tilde{\Omega}_{1}$$
(14)

$$\tilde{u}_1(\sigma - m\Omega_0) = i2r\Omega_0\tilde{\Omega}_1 - i\frac{C_{s_0}^2}{\Sigma_0}\tilde{\Sigma}_1' - i\tilde{\psi}_1'$$
(15)

$$\tilde{\Omega}_1(\sigma - m\Omega_0) = -i\frac{\kappa_0^2}{2r\Omega_0}\tilde{u}_1 - \frac{c_{s_0}^2}{\Sigma_0}im\tilde{\Sigma}_1 + \frac{1}{r^2}im\tilde{\psi}_1$$
(16)

with the radial derivative $f' = \partial f / \partial r$.

7.2 Theory: Local analysis

- let the radial dependence given by $\propto e^{ikr}$
- assume $kr \gg m$ (tight winding approximation)
 - i.e.: radial wavelength (1/k) small against azimuthal (r/m)

$$\tilde{\Sigma}_1(\sigma - m\Omega_0) = k \Sigma_0 \tilde{u}_1 \tag{17}$$

$$\tilde{u}_1(\sigma - m\Omega_0) = i2r\Omega_0\tilde{\Omega}_1 + \frac{G_{S_0}^c}{\Sigma_0}k\tilde{\Sigma}_1 + k\tilde{\psi}_1$$
(18)

$$\tilde{\Omega}_1(\sigma - m\Omega_0) = -i \frac{\kappa_0^2}{2r\Omega_0} \tilde{u}_1$$
(19)

- Now axially symmetric perturbations (m = 0)
- and vanishing perturbation potential ψ_1 (pure Keplerian disk)

dispersion relation :
$$\sigma^2 = \kappa_0^2 + c_{s_0}^2 k^2$$
 (20)

- Sound waves and epicyclic oscillations.
- Stable for $\kappa_0^2 > 0$ (Rayleigh-Criterion), then $\sigma^2 >$ for all k.

7.2 Theory: Potential perturbation

For self-gravity the disk potential ψ follows from ψ the Poisson-equation

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\psi}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2\psi}{\partial\varphi^2} + \frac{\partial^2\psi}{\partial z^2} = 4\pi G\Sigma\,\delta(z) \tag{21}$$

The perturbed potential ψ_1 is given with the above approx. by the perturbed Poisson-equation

$$\frac{\partial^2 \psi_1}{\partial r^2} + \frac{\partial^2 \psi_1}{\partial z^2} = 4\pi G \Sigma_1 \,\delta(z) \tag{22}$$

Above and below the disk the right hand side vanishes, and with $\psi \propto e^{ikr}$ it follows

$$-k^{2}\psi_{1} + \frac{\partial^{2}\psi_{1}}{\partial z^{2}} = 0 \implies \psi_{1} \propto e^{ikr - |kz|}$$
(23)

Where we used the absolute value |kz| for symmetry reasons. Vertical integration of eq. (22) yields

$$\left. \frac{\partial \psi_1}{\partial z} \right|_{-\epsilon}^{+\epsilon} = 4\pi G \Sigma_1 \tag{24}$$

7.2 Theory: Dispersion relation I

With $\frac{\partial \psi_1}{\partial z} = -|k|\psi_1$ it follows

$$\tilde{\psi}_1 = -\frac{2\pi G\tilde{\Sigma}_1}{|k|} \tag{25}$$

Substituted in Eq. (17-19) it follows

$$(\sigma - m\Omega_0)^2 = \kappa_0^2 + c_{s_0}^2 k^2 - 2\pi G |k| \Sigma_0$$

I.e. rotation (κ_0) and pressure (c_s) cause stabilisation, but self-gravity (Σ_0) a **destabilisation**!

Consider now **axially symmetric** perturbations (m = 0). stability $\sigma^2 > 0$ (σ real). Marginal stability $\sigma = 0$.

Substitute now $\sigma = 0$ and divide Eq. (26) by κ_0^2

$$0 = 1 + \frac{Q^2}{4} \left(\frac{|k|}{k_T}\right)^2 - \frac{|k|}{k_T}$$
(27)

With

$$\kappa_T \equiv \frac{\kappa_0^2}{2\pi G \Sigma_0} \quad \text{und} \quad Q \equiv \frac{\kappa_0 C_{s_0}}{\pi G \Sigma_0}$$
 (28)

(26)

7.2 Theory: Dispersion relation II

or with $\zeta = \lambda/\lambda_T \equiv k_T/|k|$ it follows for stability (rhs side of Eq. (26) or (27) > 0) $\zeta^2 - \zeta + \frac{Q^2}{4} \ge 0$ oder $Q^2 \ge 4(\zeta - \zeta^2)$ (29)

stabilized
stabilized
by pressure

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In a self-gravitating disk the dispersion relation for axisymmetric disturbances reads

$$\sigma^2 = \kappa_0^2 + c_{s_0}^2 k^2 - 2\pi G |k| \Sigma_0$$
(30)

'plus' sign: stable oscillations 'minus' sign: destabilization.

From this it follows that an axially symmetric flow is stable for

$$Q \equiv \frac{c_s \kappa_0}{\pi G \Sigma_0} > 1 \qquad \text{(Toomre-Criterion)} \tag{31}$$

- Pressure and rotation stabilise, gravitation destabilises
- Important in galactic dynamics
- non-axially symmetric: higher Q
- in protoplanetary disks: $\kappa_0 = \Omega_0 = \Omega_K$

7. Planets formed by self-gravity: Organisation

Lecture overview: 7.1 Introduction 7.2 Theory 7.3 Disk fragmentation 7.4 Planet evolution

(Zhu ea 2012)

7.3 Disk fragmentation: Massive disks

Follow the fate of unstable disks through non-linear hydro-simulations

Typical behaviour of a massive self-gravitating disk:

Non-axially symmetric disturbances with spiral arms (cf. galaxies)

In Fig. on right: Disk with $M_{\text{disk}} = 0.07 M_{\odot}$ around star with $M_* = 0.5 M_{\odot}$ with $t_{\text{cool}} = P_{\text{out}}(R_{\text{out}})$ Physical extension 120AU (displayed: T_{eff} , similar to ρ)



7.3 Disk fragmentation: Connection to Solar System

A sufficient mass rich (or cool) disk is gravitationally unstable for (Toomre-Criterion)

$$Q = rac{c_{s}\Omega}{\pi G\Sigma} < Q_{
m crit} = 1$$
 (32)

Consider now disk at 10AU with $H/r \approx 0.05$ (i.e. $c_s \simeq 0.33$ km/s). For Q = 1 we require

$$\Sigma\simeq 10^3 g/cm^2$$

This is much larger than the MMSN (Minimum Mass Solar Nebula).

For the Solar System the gravitational instability could only have worked in an earlier phase, when the disk mass was still high.

The mass of such a fragment would be

$$M_{
m
ho} \sim \pi \Sigma \lambda_{
m crit}^2 = rac{4\pi c_{
m s}^4}{G^2 \Sigma} \sim 2 M_{
m Jup}$$

So: in principle gas giant with suitable mass could formed! (The idea goes back to Kuiper (1951) or Cameron (1978))

7.3 Disk fragmentation: Instability region

Consider viscous accretion disk with constant $\dot{M} = 3\pi\nu\Sigma$, and $\nu = \alpha c_s H = \alpha c_2^2/\Omega$

$$\Rightarrow Q \propto \frac{c_s^3}{\dot{M}}$$

sound velocity drops outward.

The most unstable region is in the outer parts of the disk

Heating by external sources will influence the stability of the disk

In Fig. on right Disk with $\approx 160 M_{Jup}$

• local isothermal
$$\gamma = 1$$

$$\circ$$
 local adiabatic $\gamma=$ 1.4

(
$$\pmb{p}\propto
ho^{\gamma}$$
)



7.3 Disk fragmentation: Example simulation

SPH-Simulation 200,000 particles, $Q_{\rm min} \simeq 2$ Disk initially locally isothermal (H/r = const) $T_{\rm out}(20AU) = 100K$ Top left: No cooling, after 350 yrs - smooth density - no fragmentation - Q > 1 everywhere in disk Then: switch cooling on

 $\mathit{t}_{cool} = 0.2\,\mathrm{K}\,/\mathrm{yr}$

(constant cooling rate) Snapshots at 450, 550, 650 yrs Fragmentation if $T \le$ 42K (then Q < 1)



(Mayer et al. 2004)

7.3 Disk fragmentation: Cooling/Heating I

Local instability (fragmentation) is determined by heating and cooling of the disk

- If cooling higher \Rightarrow instability
- if heating higher \Rightarrow Stability

Heating processes:

- internal shock waves (Spiral arms, Shock-dissipation)
- viscosity (α -disks, visc. dissipation)
- external heating (by central star & nearby stars, important in outer parts)

Cooling processes:

- through equation of state: e.g. locally isothermal
- simple cooling laws
- radiative cooling (from disk surfaces)

Cooling time $t_{cool} = e_{therm}/(de_{therm}/dt)$ is *control parameter*, that determines possible fragmentation. Local stability analysis gives (Gammie, 2001)

$$t_{\text{cool}} \leq 3\Omega^{-1} \Rightarrow \text{fragmentation}$$
 (33)
 $t_{\text{cool}} \geq 3\Omega^{-1} \Rightarrow \text{no fragmentation}$ (34)

Simple estimate:

in a thin, stationary (viscous) accretion disk cooling & heating balance exactly (Pringle, 1981)

$$rac{e_{ ext{therm}}}{t_{ ext{cool}}} = \Sigma
u \left(rac{d\Omega}{dr}
ight)^2 \qquad \Rightarrow \qquad t_{ ext{cool}} \simeq rac{4}{9} rac{1}{\gamma(\gamma-1)lpha} \, \Omega^{-1}$$

For $\alpha \sim 10^{-2}$, $\gamma = 1.4$ we get $t_{cool} \sim 12$ periods. ($e = \Sigma c_v T$ and $\nu = \alpha c_s H$) (approx. timescale for changes of the thermal structure of an acc-disk) often used: β -cooling $t_{cool} = \beta \Omega^{-1}$

Additional complications:

- convection in disk (Efficiency of radiative transport)
- efficiency of turbulence (Magneto-Rotational-Instability, Dead-zones)
- Chemical composition (Opacity)
- External influences, e.g. passing star ('Triggering' of an instability)
- Stability of the fragments against shear flow in the disk

Perform parameter studies using numerical simulations

7.3 Disk fragmentation: Compare 4 numerical methods

In principle 2 alternative methods SPH

Smoothed-Particle-Hydrodynamics

Grid-Codes

finite differences, finite volume, Riemann-solver ...

Details often depend on numerical parameter:

- artificial viscosity
- resolution
- (grid points, particle number)
- Selfgravity

(solver, smoothing length, ..)

Important: compare different methods



Indiana Code (Cyl.Grid) FLASH (AMR-Cart.Grid) (Durison et al. PPV, 2007)

7.3 Disk fragmentation: Numerics - resolution

3D SPH-simulations(F. Meru, 2010)hydrodynamics: $M_{Star} = 1 M_{\odot}$, $M_{Disk} = 0.1 M_{\odot}$ Only artificial viscosity and β -cooling: $\beta = t_{cool}\Omega$, $0.25 \le r \le 25$ Vary particle number (32,000 to 16 mil.)at t = 5.3, 6.4, 5.3, 2.5 ORPfragmented \blacktriangle ; not \Box , borderline \bigcirc



W. Kley Planet Formation, Gravitational Instability, 45th Saas-Fee Lectures, 2015

7.3 Disk fragmentation: Numerics - smoothing

2D grid-based simulationen (FARGD) (smoothing considers vertical extent of disk)



Realistic smoothing: $\epsilon \approx H \Rightarrow$ fewer fragments

7.3 Disk fragmentation: Animation I

Evolution of a self-gravitating protoplanetary disk

(Durison et al., 2005)



7.3 Disk fragmentation: Animation II

Gravitational instability in protoplanetary disk Hydrodynamics: $M_{Star} = 1 M_{\odot}$, $M_{disk} = 0.1 M_{\odot}$ Locally isothermal

(L. Mayer, 2000)



2D grid simulations

Hydrodynamics: $M_{Star} = 1 M_{\odot}$, Viscous heating and radiative cooling

(Tobias Müller, 2010)



7.3 Disk fragmentation: Interaction with particles

Particles that are embedded in the gas experience a (hydrodynamic) drag They move relative to the gas into the direction of the pressure maximum here: acccumulation in the spiral arms

 \Rightarrow support of the instability, and enrichment with metalls (e.g. for cores) combination of core instability and gravitational instability



7. Planets formed by self-gravity: Organisation

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(Zhu ea 2012)

7.4 Evolution: 2D radiative simulations



Fragmentation is possible in principle but 2 challenges

- Mass challenge: disk must be massive to fragment ($M_d > 0.3M_{\odot}$) high infall rate, and fragment grow fast \rightarrow end up as brown dwarfs BD
- Migration challenge: fast inward migration < 1000yrs. Only large masses survive, small ones are tidally disrupted

GI leads to massive BDs or binaries

7.4 Evolution: 3D radiative simulations



Initial conditions according to cloud collapse Thermal evolution determined by mass accretion, hot fragments Massive loss of fragments into center Massive objects with $\gtrsim 0.06 M_{\odot}$ form

 i.e. same findings as before

7.4 Evolution: Fate of the clumps

Simulations show: Large mass and rapid migration. What about dust accretion & core formation ?



Plot: Dust mass in a fragment that reached T > 1000K. No settling and dust to gas ratio 1/100.

Need a few 10⁴yrs to form a core of a few Earth masses.

Time could possibly be shortened by pebble accretion (Boley ea 2012)

Additional issues:

Clumps that reach the inner regions are disrupted and may lead to accretion events such as FU Ori outbursts (Vorobyov & Basu, 2006). Upon inward migration clumps might lose their envelopes by tidal effects and remain as rocky planets (tidal downsizing scenario) (Nayakshin, 2010).

Investigate the evolution of planets in massive, gravo-turbulent disks with β -cooling ($\beta = t_{cool}\Omega$) that are stable with respect to fragmentation (Baruteau ea 2011).



Planets migrate inward very rapidly, with a speed equivalent to type I migration. No time for gap opening.

7.4 Evolution: Hot vs. Cold start

How to distinguish between Core Accretion planets and Gravitational Instability planets

Cold start refers to core accretion models (gas in disk can cool efficiently), while disk instability leads to rapid collapse where the planet remains hot for some extended time.



Plot:

Evolution of effective temperature for giant planets with 1, 2, 5 and 10 times Jupiter's mass.

Red: GI planets Blue: CA planets (Spiegel & Burrows, 2012). Planet formation via the GI-channel:

- depends crucially on the heating/cooling balance only possible in outer disk
- **need** massive disks ($M_d \sim 0.3M_*$) to fragment (with infall)
- formed fragments migrate inward rapidly
- formed fragments get easily tidally disrupted
- Scenario probably not suited to form HR 8799

Observations show that distant planets may be rare (only very few secured systems, HR8799, β Pic, or GJ 504). Does this mean that the main production channel appears to be Core Accretion?