## Multi-body systems

Wilhelm Kley
Institut für Astronomie \& Astrophysik \& Kepler Center for Astro and Particle Physics Tübingen

> EBERHARD KARLS
> UNIVERSITAT TUBINGEN

March 2015

## 6. Multi-body systems: Organisation

Lecture overview:
[.651: Resonances

## .6.2 Dynamics

6.3 Systems .
6.4 Circumbinary planets

### 6.1 Resonances: Example from the Solar System

Orbital Periods: Neptune: 165yrs, Pluto: 248yrs Semi-major axis: Neptune: 29.7 AU, Pluto: 35.5 AU

(NASA: New Horizon Mission)

Eccentricity $e_{p}=0.25$ crossing orbits
Stabilisation by: 3:2 Resonance
Neptune: 3 Orbits
Pluto: 2 Orbits
Pluto shares orbits with plutinos
$\Rightarrow$ Pluto degraded from planet to dwarf planet

Pluto-Flyby on 14. July, 2015.

### 6.1 Resonances: Example of orbits


(Gravity Simulator, Tony Dunn)

Planetary orbits:
in the corotating-frame of Neptune
Pluto
Neptuns
Uranus
Saturn
Orbits avoid close encounters
Pluto maximizes its separation from Neptune
The resonant condition stabilizes Pluto's orbit

### 6.1 Resonances: Definition - Mean motion resonance

A Mean Motion Resonance (MMR) is given by an integer ratio of their mean motions $n_{i}$

$$
\begin{equation*}
n_{1} / n_{2}=p / q \quad \text { mit } \quad q, p \in N \tag{1}
\end{equation*}
$$

$n_{i}$ mean orbital velocity of $i$-th planet (1: inner, 2 : outer planet)

The resonant angles are then defined (with $p>q$ )

$$
\begin{equation*}
\phi_{k}=p \lambda_{2}-q \lambda_{1}-p \varpi_{2}+q \varpi_{1}+k\left(\varpi_{2}-\varpi_{1}\right), \tag{2}
\end{equation*}
$$

where $\lambda_{i}$ and $\varpi_{i}$ are the mean longitudes and longitudes of periapse.
The integer $k$ in Eq. (2) satisfies $q \leq k \leq p$ and has $p-q+1$ possible values. Of the $p-q+1$ resonant angles, at most two are linearly independent, and at least 1 angle will librate (resonant condition)
Example 2:1 System ( $p=2$ and $q=1$ )

$$
\begin{equation*}
\Phi_{1}=2 \lambda_{2}-\lambda_{1}-\varpi_{1}, \quad \Phi_{2}=2 \lambda_{2}-\lambda_{1}-\varpi_{2} \quad\left(\Delta \varpi=\Phi_{1}-\Phi_{2}\right) \tag{3}
\end{equation*}
$$

For details, see Book: Murray\&Dermott Solar System Dynamics

■ The resonant angles follow from an expansion of the perturbation potential between the two planets, that would move otherwise on unperturbed Kepler-orbits.
■ Expand the perturbation in a Fourier-series
■ The resonant angles are the stationary phases in the expansion
■ If an angle $\Phi_{i}$ is in libration (i.e. covers a range smaller than $[0,2 \pi])$, then the system is said to be in a Resonance
■ If both angles $\Phi_{i}$ (for a 2:1 resonance) librate (i.e. $\Delta \varpi$ as well) then the system is in apsidal corotation (ACR)

### 6.1 Resonances: Two sample systems

Animation of the observed motion of two systems

GJ 876: $P_{1}=30 d, P_{2}=60 d$

(Man Hoi Lee)
retrograde prcession
$\Delta \varpi$ in libration $(\Delta \varpi)_{\max } \approx 30^{\circ}$ apsidal corotation
Distance of periastrons: $\Delta \varpi=\varpi_{2}-\varpi_{1}$

### 6.1 Resonances: Kepler-Planets



- Higher numbers near 2:1 and 3:2 resonances
- but the systems lie slightly shifted from the resonance


### 6.1 Resonances: Formation of a resonant system

2 massive planets in disk (in HD $73526 \approx 2.5 M_{\text {Jup }}$ each)


Two planets: joint, large gap Outer planet : Pushed inward by outer disk Inner planet : Pushed outward by inner disk

Separation reduction: Resonant capture (Sandor ea. 2007)

### 6.1 Resonances: Eccentricity Pumping

Here: System-parameter of GJ 876 (damping of inner \& outer disk)


System ends in: apsidal corotation, with correct eccentricities Less disk damping: $\Rightarrow$ much higher $e \Rightarrow$ possible Instability
6. Multi-body systems: Organisation

## Lecture overview: <br> $\square$ 6.1 Resonances <br> - 6.2 Dynamics

- 6.3 Systems
- 6.4 Circumbinary planets


### 6.2 Dynamics: Eccentricity vs. distance (RV-data)



### 6.2 Dynamics: Eccentricity and Inclination

What causes the high eccentricities and inclinations ?
In principle 2 models:

- differential migration of planets in the disk:
$\Rightarrow$ compact (resonant) configuration
$\Rightarrow$ disk dissipates and reduces the stabilisation
$\Rightarrow$ instability of planetary system
$\Rightarrow$ excitation of $e$ and $i$
$\Rightarrow$ gravitational scattering processes
- by an additional, massive body (star, planet):
(tidally driven migration, by the Kozai-effect)
$\Rightarrow$ third body excites eccentricity (inclination) to high values
$\Rightarrow$ oscillation of eccentricity and inclination (of planet)
$\Rightarrow$ for very high $e \Rightarrow$ tidal interaction with central star
$\Rightarrow$ damping of semi-major axis
(not by disk-planet interaction)


### 6.2 Dynamics: Migration and scattering processes



N -body simulations with 3 or 4 planets

Start with:

- system near resonance

Consider:

- Star-Planet interaction

Black: Dots \& circles Planets after $10^{9} \mathrm{yrs}$ In brown - Kreise: Objekts after $10^{8} \mathrm{yrs}$ In orange - squares: Obsevations

For more details: (Beauge \& Nesvorny 2012) (Juvic \& Tremaine 2008)

### 6.2 Dynamics: Kozai-Mechanism

A main-belt asteroid in the Solar System moves under the influence of the Sun and Jupiter. If the initial inclination of the body is higher than $i_{0}>i_{\text {crit }}=38^{\circ}$ the orbits will become highly eccentric (Kozai, 1962). Sometimes also Lidov-Kozai mechanism.


Consider a hierarchical restricted 3-body system

For asteroid:

$$
\begin{aligned}
& m_{0}=\text { sun } \\
& m_{1}=\text { asteroid } \\
& m_{2}=\text { Jupiter }
\end{aligned}
$$

for proto planet:
$m_{0}=$ star
$m_{1}=$ planet
$m_{2}=$ secondary star
(Wen, 2003)
For the satellite, the orbit averaged equations of motion have a conserved quantity, $L_{z}=\cos (i) \sqrt{1-e^{2}}, z$ is perpendicular to the orbit of the binary.

### 6.2 Dynamics: Kozai-Citations

Secular perturbations of asteroids with high inclination and eccentricity Kozai, Astronomical Journal, 1962

Citations/Publication Year for 1962AJ.....67..591K


### 6.2 Dynamics: The planetary system HD 80606



HD 80606
a highly eccentric planetary system

$$
\begin{aligned}
& P_{p}=111 \mathrm{~d} \\
& m_{p}=3.94 \mathrm{M} \text { Jup } \\
& \mathrm{a}_{\mathrm{p}}=0.449 \mathrm{AU} \\
& \mathrm{e}_{\mathrm{p}}=0.933
\end{aligned}
$$

Each grid square $=0.1 \mathrm{AU} \times 0.1 \mathrm{AU}$
Planet and star not drawn to scale
(Wikepedia)

### 6.2 Dynamics: Kozai mechanism for HD 80606

Consider the motion of the planet in the HD 80606 system under the action of an additional secondary star at large distance

Todays parameter: $m_{p}=3.94 M_{\text {Jup }}, a_{p}=0.449 \mathrm{AU}, e_{p}=0.933$
start with: $a_{p}=5 \mathrm{AU}, \mathrm{i}=85.6^{\circ}, \mathrm{e}=0.1$
binary star parameter: $m_{\text {bin }}=1.1 \mathrm{M}_{\odot}, a_{\text {bin }}=1000 \mathrm{AU}, \mathrm{e}_{\text {bin }}=0.5$
(e.g. Models by: Wu \& Murray 2003, Fabrycky \& Tremaine 2007)

(Daniel Fabrycky, Harvard)

### 6.2 Dynamics: Orbital elements of HD 80606





Parameter:
Start Today

| $\mathrm{a}_{\mathrm{p}}(\mathrm{AU})$ | 5 | 0.449 |
| :--- | ---: | ---: |
| e | 0.1 | 0.933 |
| i | $85.6^{\circ}$ | $50^{\circ}$ |

over a timescale of 3 Ga the $\diamond$ denotes the present position (Fabrycky \& Tremaine, 07)

## 6. Multi-body systems: Organisation

## Lecture overview:

- 6.1 Resonances
6.2 Dynamics
6.3 Systems


### 6.4 Circumbinary: Some observed Systems

## Observed circumbinary planets

 (orbits normalized to the instability region)

Presently 8 Kepler systems known. Orbits are unstable if $a_{p}$ is smaller than 2-5 times $a_{b i n}$ (Holman \& Wiegert, 1999)

### 6.4 Circumbinary: Evolution of planets

What brings the planets so close to the instability limit?
Assume planets have formed further out in pp-disk (flat systems)
(not possible to form the in situ)
Study subsequent evolution of planets in disk (migration)
Analyse two cases: Kepler-38 and Kepler-34
(See work by Pierens \& Nelson (2013))

### 6.4 Circumbinary: Kepler 38: System Architecture



Binary Parameter:
(Kepler 38A,B)
$M_{1}=0.95 M_{\odot}$,
$M_{2}=0.25 M_{\odot}$
$P_{B}=18.6$ days
$a_{B}=0.15 \mathrm{AU}$
$e_{B}=0.10$

Planet Parameter:
(Kepler 38b)
$m_{p}=0.36 M_{\text {Jup }}$
$P_{p}=105.6$ days
$a_{p}=0.46 \mathrm{AU}$
$e_{p}=0.03$

### 6.4 Circumbinary: Kepler 38 - Circumbinary Disk Model

Two-dimensional disk around central binary, hydrodynamics, $\alpha$-viscosity no self-gravity. Isothermal and radiative disks

Animation of disk around Binary Star (shown is surface density of inner disk)

(Kley \& Haghighipour, 2014)

### 6.4 Circumbinary: Kepler 38 - Evolution of surface density



Initial profile:

$$
\Sigma \propto r^{-1 / 2}
$$

## Equilibrium: after $\approx 3000 \mathrm{yrs}$

Outer Boundary: $\Sigma$ fixed

Inner Boundary:
Open

## Kepler 38: Migration of planet in disk



Planets started at different distances, in original/relaxed disks All stop at same distance: at $\approx 0.43 \mathrm{AU}$

## Kepler 38: Surface density with planet

Planet is 'parked' at inner edge of disk


## Kepler 38: Animation: Planet in disk

Animation of Disk and Planet around Binary Star (shown is the final phase, in 5:1 resonance)


- 5:1 probable unstable upon disk dissipation
■ some planet evolutions are unstable
■ inside observed location and too high eccentricity
- vary disk properties to achieve stopping further out
- and use radiative disks


## Kepler 38: Summary/Outlook

Results: Kepler 38:
■ Circumbinary planets can be explained by a migration process
■ Planets are 'parked' near gap edge, just outside of instability region
■ Exact location depends on disk physical parameter
Open issues:

- Disk structure not clear - even without planet

■ Todo: Parameter variation, radiative disks, 3D simulations

## Kepler 34: Eccentric binary, System Parameter

Comparison: Kepler 38 and Kepler 34 Binary Parameter:

|  | (Kepler 38) | (Kepler 34) |
| :--- | :--- | :--- |
| $M_{1}\left[M_{\odot}\right]$ | 0.95 | 1.05 |
| $M_{2}\left[M_{\odot}\right]$ | 0.25 | 1.02 |
| $P_{B}[$ days $]$ | 18.6 | 28 |
| $a_{B}[\mathrm{AU}]$ | 0.15 | 0.23 |
| $e_{B}$ | 0.10 | 0.52 |

Planet Parameter:

|  | (Kepler 38) | (Kepler 34) |
| :--- | :--- | :--- |
| $m_{p}\left[M_{\text {Jup }}\right]$ | 0.36 | 0.21 |
| $P_{p}[$ days $]$ | 105.6 | 288 |
| $a_{p}[\mathrm{AU}]$ | 0.46 | 1.09 |
| $e_{p}$ | 0.03 | 0.18 |

## Kepler 34: Disk Structure

Surface density and eccentrictiy of disk with no planet



Differences to low eccentric binary:

- larger inner hole
- spike in density

■ high disk eccentricity

## Kepler 34: Animation: Planet in disk

Animation of Disk and Planet around Kepler 34 (shown is the final phase)


Summary: eccentric binary
■ Large (eccentric) hole in center of disk

- Planet stops too far out

■ Vary disk parameter ?
■ Multiple planets ?
6. Multi-body systems: Organisation

## January 2014



### 6.3 Systems: Systems of super-earths: Examples

Super-earth systems with $R_{p}<1.5 R_{\text {earth }}$ in comparison to the Solar System KOI-2859 ○ ○○ ○

| kol-2732 |  |  | Top: 8 Kepler can <br> kol-2722 |  |
| :--- | :--- | :--- | :--- | :--- |
| date systems |  |  |  |  |

### 6.3 Systems: Systems of super-earths: Characteristic

Some characteristics of Hot-super Earth systems

- terrestrial-like exoplanets radii between Earth and Neptune often in multiple systems
- extremely common $30-50 \%$ of Sun-line stars (FGK) host at least one planet with mass less than $10 M_{\text {earth }}$ with period less than 50-100 d. Probability around M -stars at least as high
- orbits
typically compact systems on non-resonant orbits orbital separations comparable to Solar System planets, in terms of Hill-radii


### 6.3 Systems: Systems of super-earths: Formation

Some suggested formation scenarios

1) in situ formation in massive disk
2) accretion during inward type I migration
3) sheparding by interior mean-motion resonances of inwardly migrating massive planets
4) sheparding by interior secular resonance inwardly migrating massive planets
5) circularisation of high-eccentric planets by tidal interactions with star
6) photo-evaporation of close-in giant planets

Numbers 3) to 6) can be ruled out theoretically and observationally (Raymond ea. 2014)

### 6.3 Systems: Systems of super-earths

## Conflicts:

3)-4) require giant planet to drive planets inward and an additional disk to damp eccentricity in both scenarios a giant planet should be present just outside of outmost planet which is not obeserved
5) circularization in principle possible (see above) but require large initial eccentricities that would result in single-planet systems, which is in conflict with observations
6) evaporation is possible for very close in planets, requires Gyr to operate. Strongly depends on distance from central star. Would only allow the innermost planet to be evaporated.

Hence 1) (in situ) and 2) (inward type I migration) are the leading contestors
(Haghighiour 2013; Raymond ea. 2014)

### 6.3 Systems: Systems of super-earths

Comments on scenarios

1) in situ

Requires migh mass ( $\approx 20-40 M_{\oplus}$ ) within fraction of an $A U$
within a MMSN context: $\Sigma=\Sigma_{0}(r / A U)^{-x}$, need steep powerlaw
$x=1.6-1.7$ and 10 times higher normalisation $\Sigma_{0}$.
i.e. high total disk mass

Observations indicate shallower profile: $x \approx 0.5-1.0$
But: Simulations show that orbital properties (eccentricities, inclinations, separations) match observations
2) migration

Simulations of migration super-earths show the formation of resonant chains
destabilisation $\Rightarrow$ collisions and accretion consecutive chain formation (see example below)

Possible distinction betwen models:

1) naked high-density rocks 2 ) lower density material containing ice but possible atmosphere could hide effect

### 6.3 Systems: Example formation scenario



Formation of hot super-Earths by type I migration
Top: orbital distances
Middle: mass growth Bottom: Comparision
size scaled to radius of the planet

For reference:
$a_{\text {Earth }} / a_{\text {Venus }}=1.6$
(Cossou ea. 2014)

## 6. Multi-body systems: Further Reading

■ Resonances:
Murray \& Dermott (1999), Kley et al. (2004)
■ Multiple:
Fabrycky \& Tremaine (2007)
■ Super-Earths: Haghighipour (2013), Raymond et al. (2014)

