

Planet disk interaction

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4. Planet-Disk: Organisation

Lecture overview:

- **4.1 Introduction**
- **4.2 Type I**
- **4.3 Type II**
- **4.4 Type III**
- **4.5 Stochastic**
- **4.6 Elements**

4.1 Introduction: Planet Formation Problems

- Not possible to form hot Jupiters in situ
 - disk too hot for material to condense
 - not enough material
- Difficult to form massive planets
 - gap formation
- Eccentric and inclined orbits
 - planets form in flat disks (on circular orbits)

But planets grow **and** evolve in disks:

⇒ Have a closer look at planet-disk interaction

Consider **Late Phase** of Planet Formation:

- Assume protoplanets (embryos) have already been formed
- Investigate **subsequent evolution** in disk,
consider gravitational interaction with star and disk

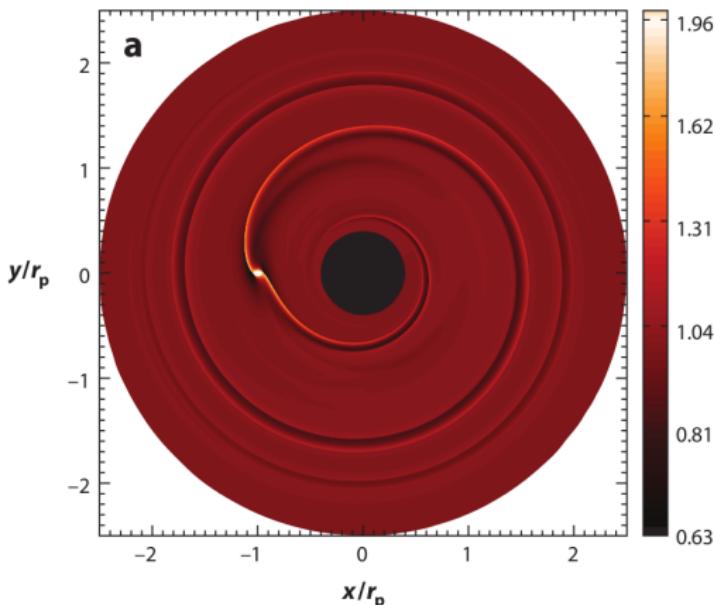
4.1 Introduction: Two main contributions

1) Spiral arms

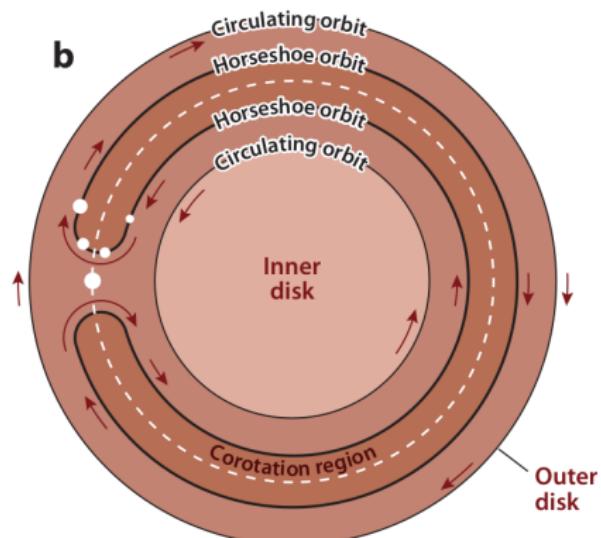
(Lindblad Torques)

2) Horseshoe region

(Corotation Torques)



(Kley & Nelson, ARAA, 50, 2012)



4.1 Introduction: Evidence for migration

- many multi-planet extrasolar planetary systems
high fraction in or close to a low-order **mean-motion resonance** (MMR)
In **Solar System**: 3:2 between Neptune and Pluto (plutinos)
- Resonant capture through convergent migration process
dissipative forces due to disk-planet interaction
- Existence of resonant systems
 - **Clear evidence for planetary migration**
- Hot Jupiters (Neptunes) & Kepler systems
 - **Clear evidence for planetary migration**

The main Problem: Migration Speed
(for Type-I and Type-II)

4.1 Introduction: Torque calculation

2 approaches:

- a) **Linearization**: for low mass planets only

basic state: axisymmetric disk, in Keplerian rotation, and given $\Sigma(r)$
add planet potential as a small disturbance of planet

linearize equations

calculate new (perturbed) disk structure

calculate torque acting on planet

- b) **Non-linear**: for all planet masses

full non-linear hydrodynamical evolution of embedded planets in disks

Will not go into details of these methods but just quote the main results

4.1 Introduction: Torques and migration

Angular momentum of planet (z -direction)

$$J_p = (mr u_\varphi)|_p = m_p r_p^2 \Omega_p \quad (1)$$

change in time by torques (**z -component**) acting on the planet

$$\dot{J}_p = \Gamma_p \quad (2)$$

Γ_p is calculated over the whole disk

$$\Gamma_p = - \int_{\text{disk}} \Sigma (\vec{r}_p \times \vec{F}) df = \int_{\text{disk}} \Sigma (\vec{r}_p \times \nabla \psi_p) df = \int_{\text{disk}} \Sigma \frac{\partial \psi_p}{\partial \varphi} df \quad (3)$$

⇒ Migration Timescale τ_M

$$\frac{1}{r_P} \frac{dr_P}{dt} = \frac{1}{\tau_M} = 2 \frac{\Gamma_p}{J_p} \quad (4)$$

Compare τ_m to evolution of disk

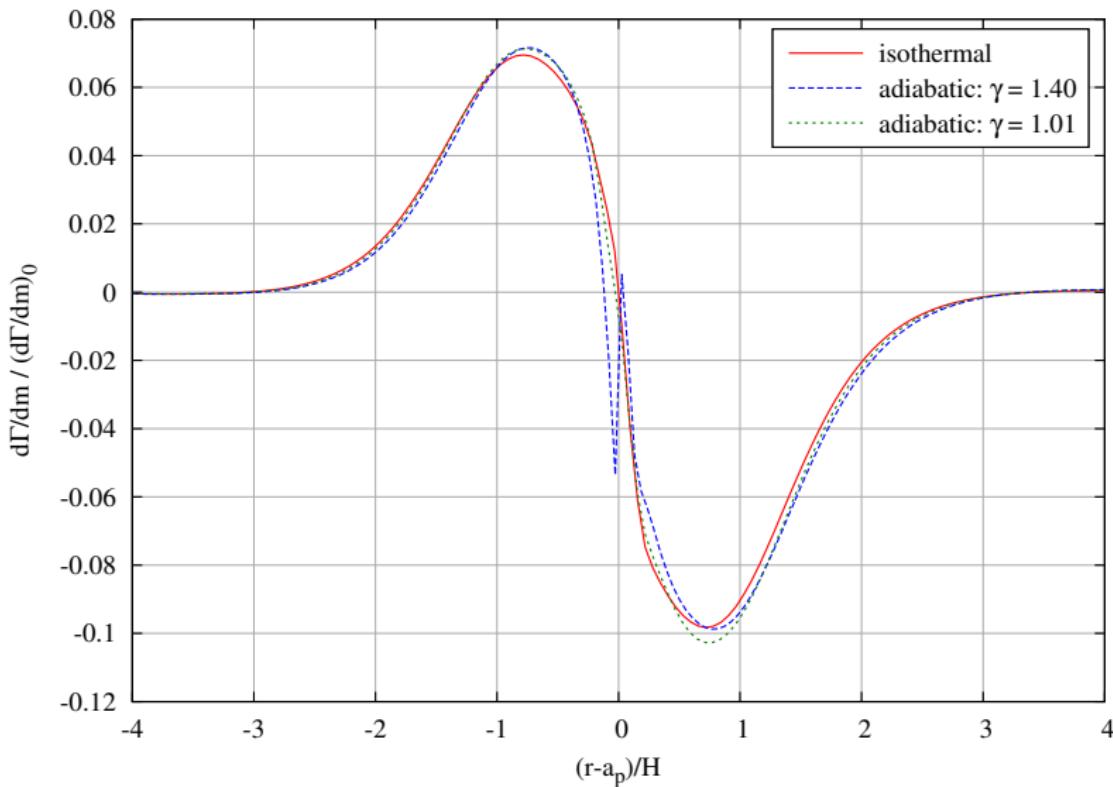
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4.2 Type I: Lindblad torque: radial torque density

$$\Gamma_{\text{tot}} = 2\pi \int \frac{d\Gamma}{dm}(r) \Sigma(r) r dr$$



4.2 Type I: The linear Torque

Results for a **locally isothermal** disk. Fixed $T(r)$.

Torque **on** the planet: (for lower mass planets, a few M_{\oplus})

3D analytical (Tanaka et al. 2002) and numerical (D'Angelo & Lubow, 2010)

$$\Gamma_{\text{tot}} = -(1.36 + 0.62\beta_{\Sigma} + 0.43\beta_T) \Gamma_0. \quad (5)$$

where

$$\Sigma(r) = \Sigma_0 r^{-\beta_{\Sigma}} \quad \text{and} \quad T(r) = T_0 r^{-\beta_T} \quad (6)$$

and normalisation

$$\Gamma_0 = \left(\frac{m_p}{M_*} \right)^2 \left(\frac{H}{r_p} \right)^{-2} (\Sigma_p r_p^2) r_p^2 \Omega_p^2, \quad (7)$$

$$\Rightarrow \text{Migration} \quad j_p = \Gamma_{\text{tot}} \quad \Rightarrow \quad \frac{\dot{a}_p}{a_p} = 2 \frac{\Gamma_{\text{tot}}}{J_p} \quad (8)$$

Classic Type I migration: Inward and fast

Too fast to be in agreement with observations (see lecture 5)

4.2 Type I: Torque Saturation

2D hydro-simulations:

- small mass
- inviscid

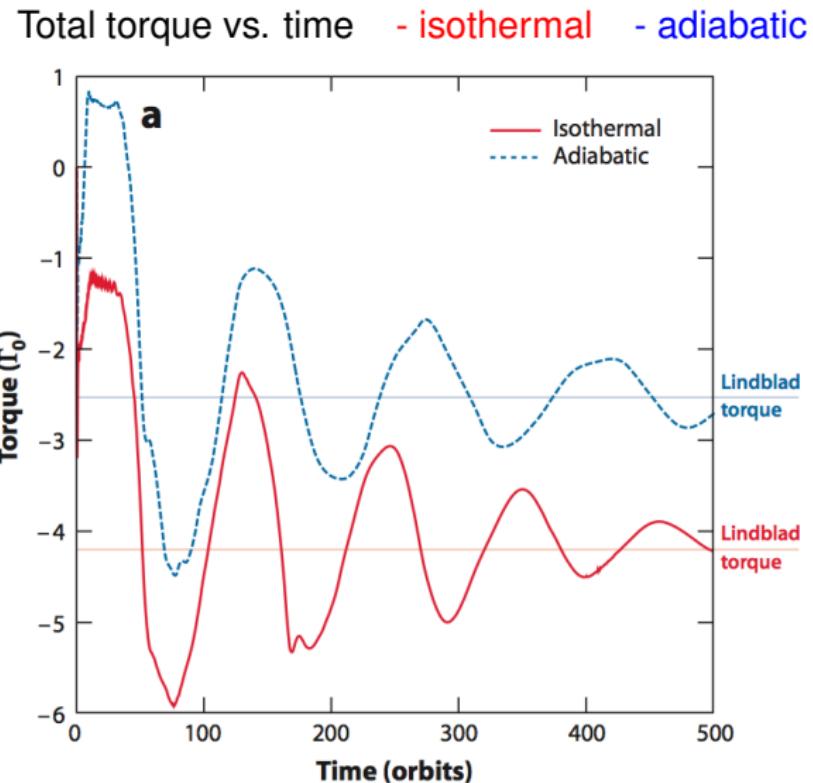
Result:

Torques are positive initially and then diminish with time and settle to the Lindblad torques (from spirals)

That means:

Corotation torques decline = saturate

How to maintain the full corotation torques ?



4.2 Type I: Torque Saturation

How to prevent corotation-torque saturation ?

From linear analysis:

Corotation torque depends on the radial gradients of the entropy, S , and vortensity $\tilde{\omega}$ where the vortensity is defined as:

$$\tilde{\omega} = \frac{\omega_z}{\Sigma} = \frac{(\nabla \times \vec{u})|_z}{\Sigma} \quad (9)$$

Now: For inviscid and barotropic ($p = p(\rho)$) flows:

$$\frac{d}{dt} \left(\frac{\omega_z}{\Sigma} \right) = 0 \quad \text{and} \quad \frac{dS}{dt} = 0 \quad (10)$$

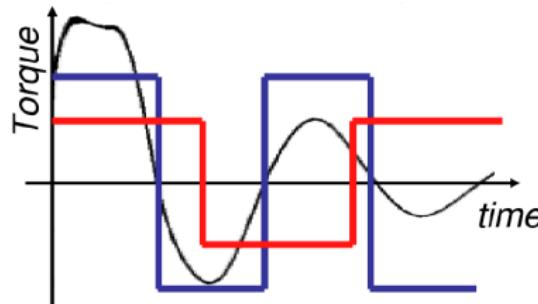
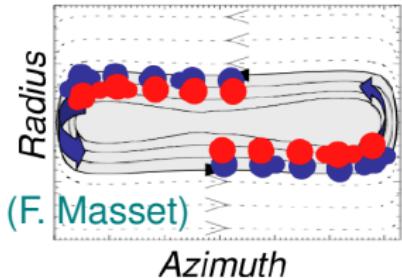
These quantities are constant along streamlines.

Initial gradients (in S and $\tilde{\omega}$) are wiped out in the corotation region

⇒ Need mechanism to maintain gradients

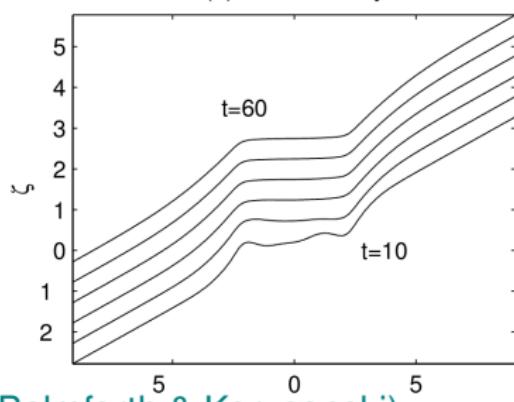
These are viscosity and radiative diffusion (or cooling)

4.2 Type I: Saturation - Origin

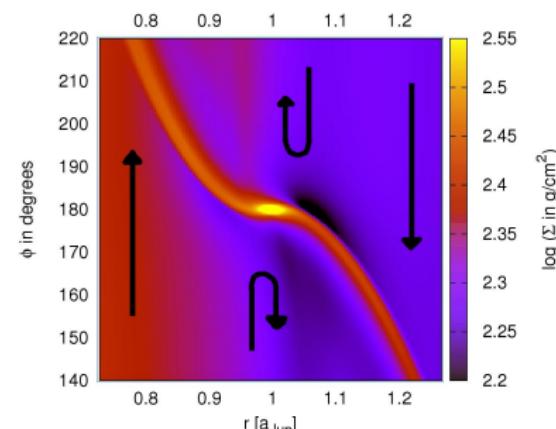


Red and Blue
Orbits have
different peri-
ods
⇒ Phase mix-
ing

(a) Mean vorticity



(Balmforth & Korycanski)

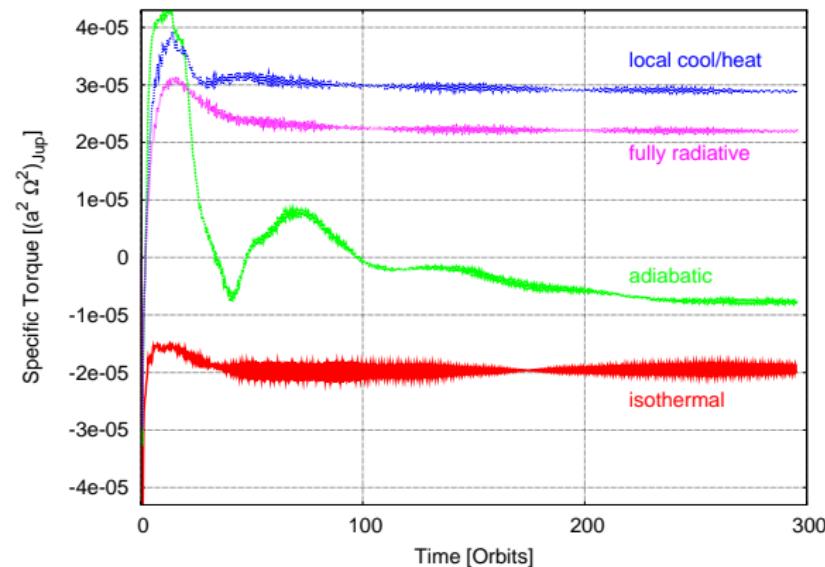


3D radiative disk (B. Bitsch)

4.2 Type I: Disk physics

$$\frac{d\Sigma c_v T}{dt} + \nabla \cdot (\Sigma c_v T \mathbf{u}) = -p \nabla \cdot \mathbf{u} + D - Q - 2H \nabla \cdot \vec{F}$$

Adiabatic: Pressure Work, Viscous heating (D), radiative cooling (Q)
radiative Diffusion: in disk plane (realistic opacities)



$M_p = 20M_{\text{earth}}$
(Kley&Crida '08)

Disk Physics determines
Direction of motion

see also:
(Paardekooper&Mellema '06
Baruteau&Masset '08
Paardekooper&Papaloizou '08)

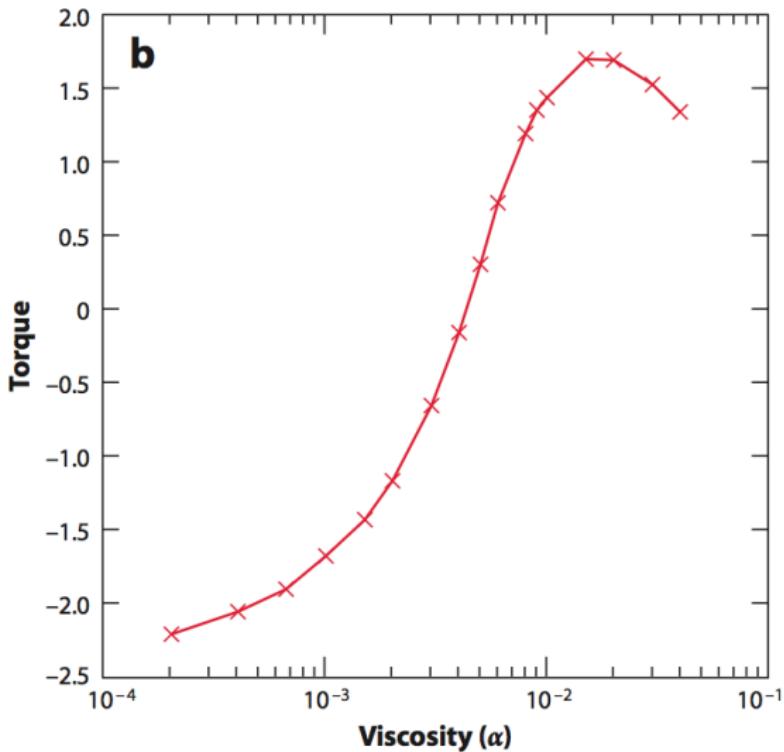
4.2 Type I: Role of viscosity

Total torque
vs. viscosity:

in **viscous** α -disk
2D **radiative** model
- in equilibrium

Efficiency depends
of ratio of timescales
 $\tau_{\text{visc}} / \tau_{\text{librat}}$
 $\tau_{\text{rad}} / \tau_{\text{librat}}$

Local disk properties
matter

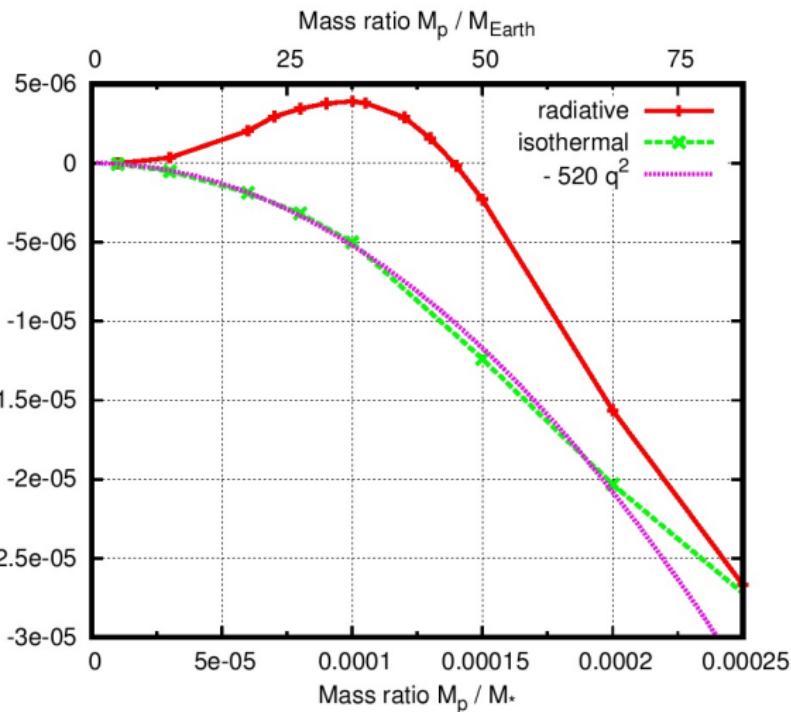


(Kley & Nelson 2012)

⇒ Need viscosity to prevent saturation !

4.2 Type I: Mass dependence

Isothermal and radiative models. Outward migration for $M_p \leq 40 M_{\text{Earth}}$



Vary M_p
(Kley & Crida 2008)

In full 3D
(Kley ea 2009)

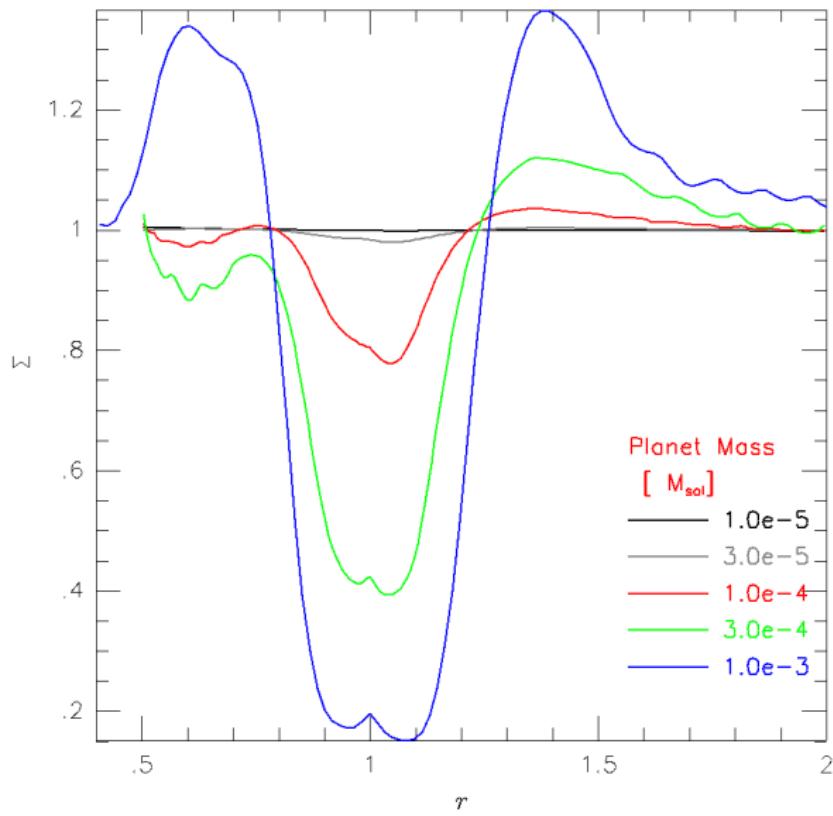
With irradiation:
(Bitsch ea. 2012)

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4.3 Type II: Gap formation: Type I \Rightarrow Type II



$$M_p = 0.01 M_{Jup}$$

$$M_p = 0.03 M_{Jup}$$

$$M_p = 0.1 M_{Jup}$$

$$M_p = 0.3 M_{Jup}$$

$$M_p = 1.0 M_{Jup}$$

Depth depends on:

- M_p
- Viscosity
- Temperature

Torques reduced :

Migration slows

Type I \Rightarrow Type II

linear \Rightarrow non-linear

4.3 Type II: Gap opening criteria

■ Thermal condition

Hill sphere $R_H = r_p(q/3)^{1/3}$ comparable/larger than disk scale height H with $q = m_p/M_*$

$$q \geq 3 \left(\frac{H}{r} \right)_p^3 = 3h_p^3 \quad (11)$$

with a disk aspect ratio $h = 0.05$: $q \geq 1.25 \cdot 10^{-4}$, or $m_p \geq 0.13m_{Jup}$

■ Viscous condition

Viscosity tends to close gap against gap opening torques. From impulse approximation (Lecture 3) we find

$$q \geq 30\pi\alpha h^2 \quad (12)$$

with $h = 0.05$ and $\alpha = 10^{-2}$: $q \geq 2.4 \cdot 10^{-3}$, or $m_p \geq 2.5m_{Jup}$

4.3 Type II: A combined gap opening criterion

Using 2D hydrodynamical viscous disk simulations, (Crida et al. 2006) find gap opening (minimum density within gap smaller 1/10 ambient density) for

$$P = \frac{3}{4} \frac{H}{R_H} + \frac{50}{qRe} \leq 1 \quad (13)$$

with the Reynoldsnumber $Re = \Omega_p r_p^2 / \nu$.

In type II regime the planet remains in the middle of the gap which moves with the disk. The disk evolves on a viscous timescale and the planet will move with the same timescale

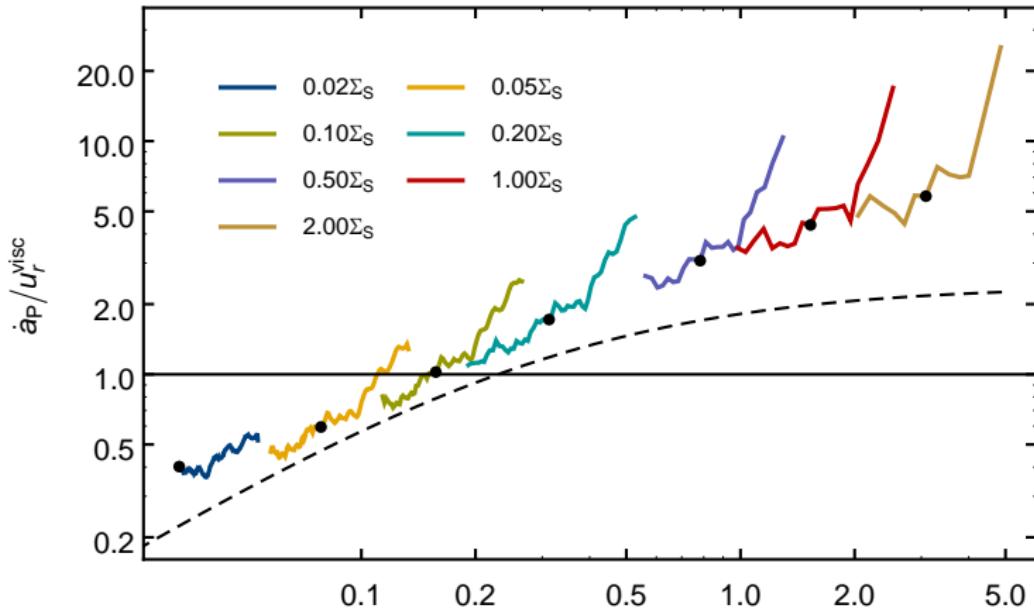
$$\tau_{mig,II} = \tau_{visc} = \frac{r_p^2}{\nu} = \frac{r_p^2}{\alpha c_s H} = \frac{1}{\alpha h^2 \Omega_p} \quad (14)$$

Question:

How well is this assumption really satisfied?

4.3 Type II: New simulations

2D hydro simulations, with net mass flow through the disk. Planet migrates in disk. Different colours \equiv different \dot{M}_{disk} . Black dot: planet at $r = 0.7r_0$. Compare migration rate to the viscous speed. Can be smaller/larger than viscous speed. Σ_S : reference density for $\dot{M} = 10^{-7} M_\oplus/\text{yr}$.

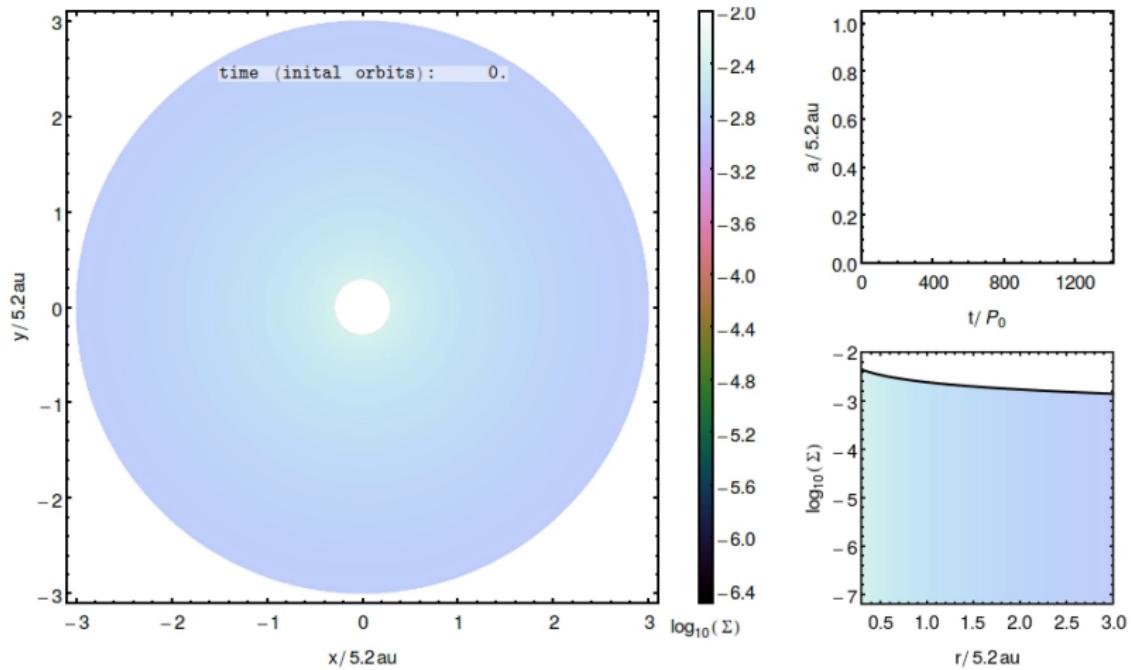


(Dürmann & Kley, 2015)

$\Sigma_P a_P^2/M_J$

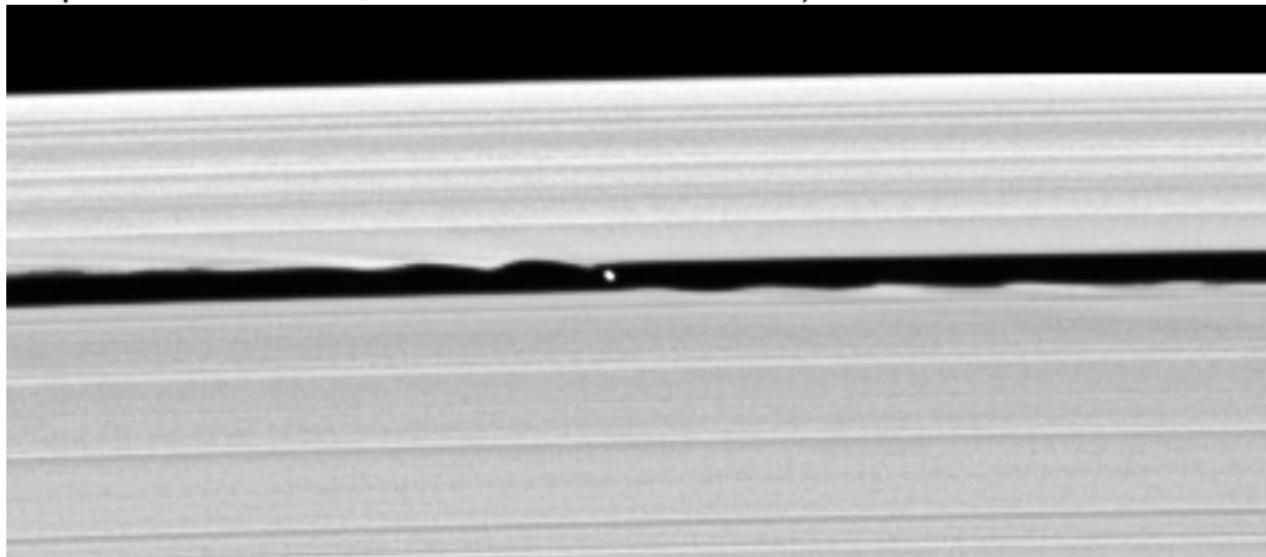
4.3 Type II: The migration process

Result implies that during the migration process material can cross the gap and is transferred from one side to the other



4.3 Type II: Evidence gap formation

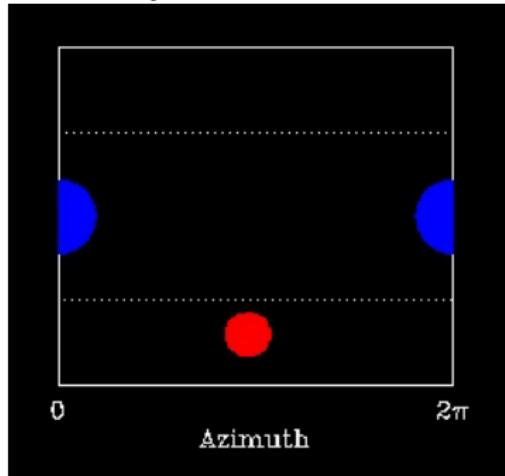
Moon (S/2005 S1) in Keeler Gap, Cassini (1. May 2005)
Gap width \approx 40 km, Moon-diameter \approx 7 km)



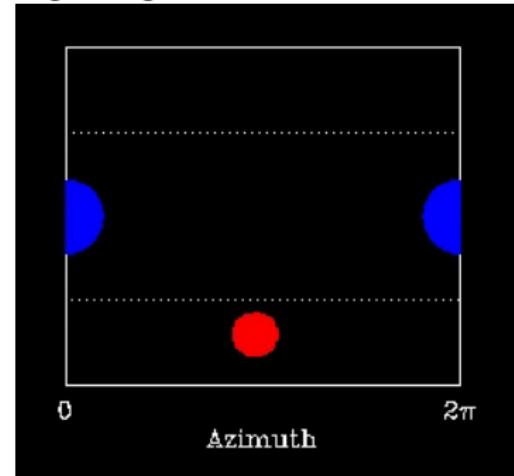
Here: very clean gap, no pressure, no viscosity

4.4 Type III: Type-III migration principle

Stationary Planet



Migrating Planet



(F. Masset, 2002)

Inner material crosses horseshoe region: gains ang.mom.

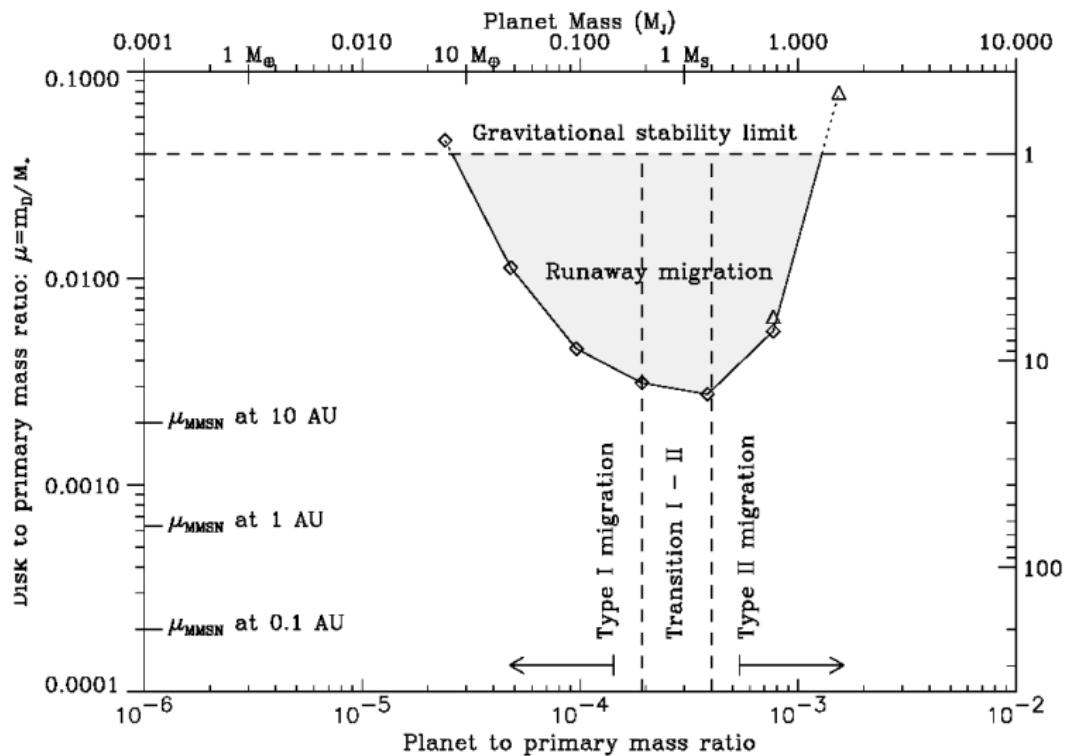
Planet loses ang.mom. \rightarrow Runaway

Need massive disk and large radii

(Masset & Papaloizou, 2003; P. Artymowicz)

Here inward, but outward same argument.

4.4 Type III: Regime



(Masset & Papaloizou, 2003) Need massive disk and large radii.

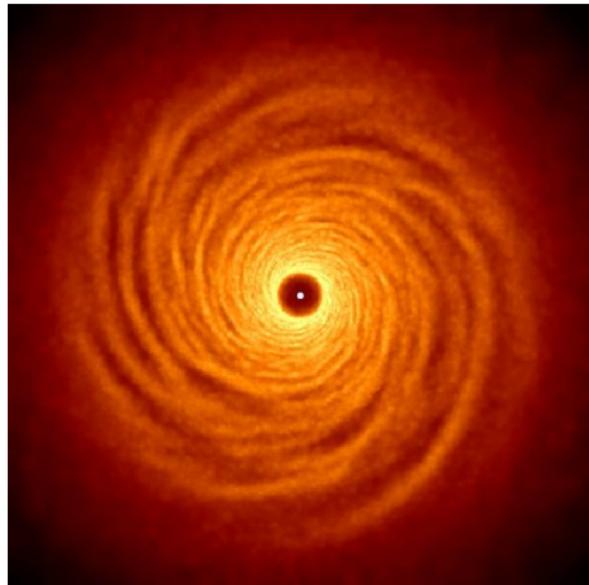
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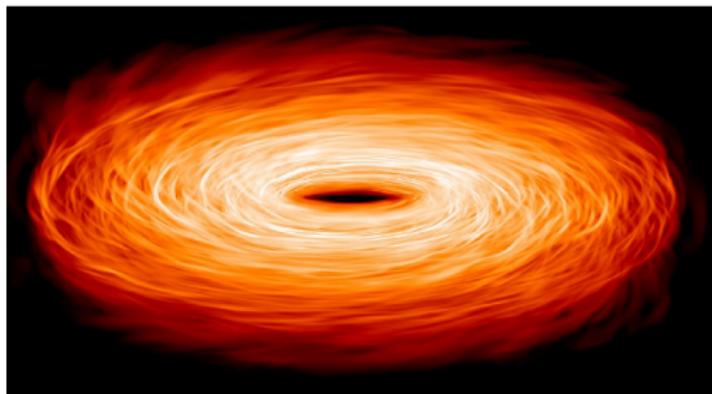
4.5 Stochastic: Turbulent disks

self-gravitating disk



(G. Lodato)

MRI turbulent disk

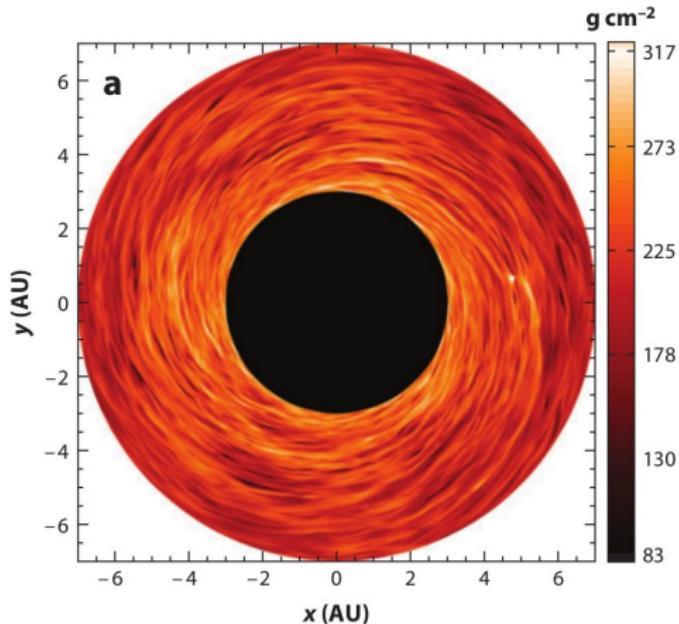


(M. Flock)

4.5 Stochastic: Planets in turbulent disks

Disks are MHD turbulent and embedded planets will feel random (stochastic) perturbations due to the density fluctuations.

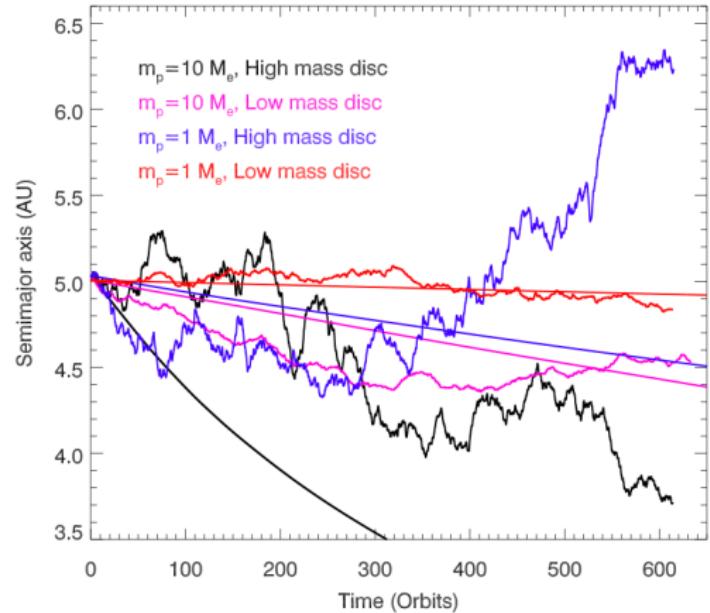
Analyze impact on migration



planet with $m_p = 10M_{\text{earth}}$
in unstratified turbulent disk
with $H/r = 0.05$
and toroidal \vec{B} -field
plasma- $\beta = p_{\text{gas}}/p_{\text{mag}} = 50$
with effective $\alpha = 0.01$

(R. Nelson)

4.5 Stochastic: Planets in turbulent disks



evolution of semi-major axis
smooth lines show migration in
equivalent laminar disks

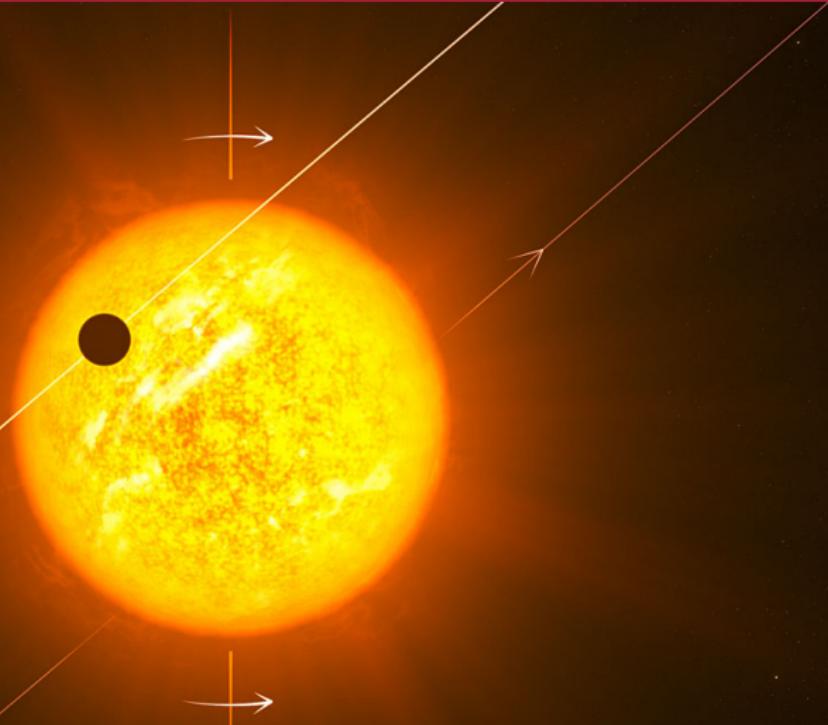
On very long time scales similar
to laminar cases

(R. Nelson)

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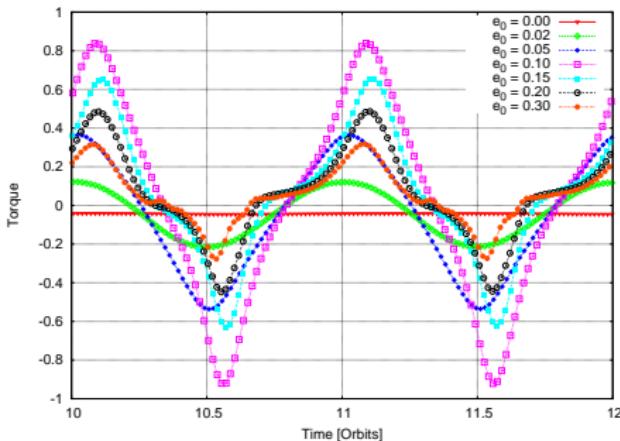
(ESO April 2010: *Turning Planetary Theory Upside Down*)

4.6 Elements: Planets on eccentric Orbits

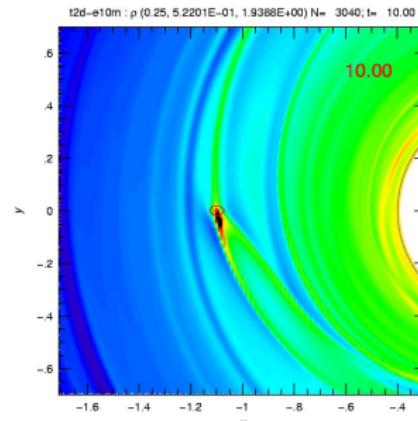
Torque on planet due to disk

Power: Energy loss of planet

$$T_{disk} = - \int_{disk} (\vec{r} \times \vec{F}) \Big|_z \, df$$



$$P_{disk} = \int_{disk} \dot{\vec{r}}_p \cdot \vec{F} \, df$$



$$L_p = m_p \sqrt{GM_* a} \sqrt{1 - e^2}$$

$$\frac{\dot{L}_p}{L_p} = \frac{1}{2} \frac{\dot{a}}{a} - \frac{e^2}{1 - e^2} \frac{\dot{e}}{e} = \frac{T_{disk}}{L_p}$$

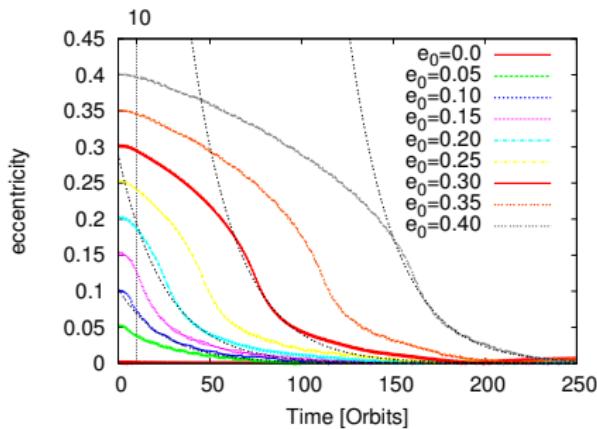
$$E_p = -\frac{1}{2} \frac{GM_* m_p}{a}$$

$$\frac{\dot{E}_p}{E_p} = \frac{\dot{a}}{a} = \frac{P_{disk}}{E_p}$$

4.6 Elements: Eccentric planets in 3D radiative disks

Planet mass $M_p = 20M_{\text{Earth}}$

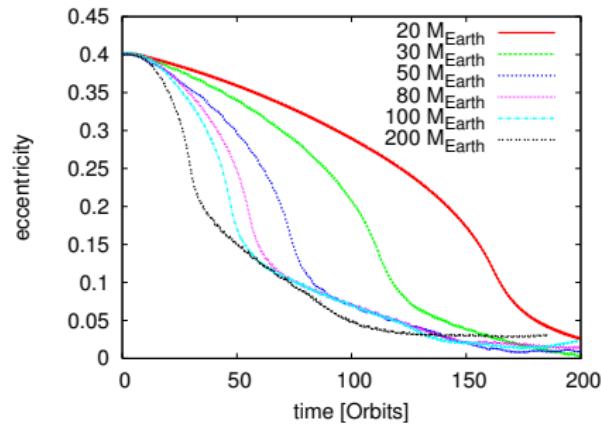
- Vary Eccentricity



(Bitsch & Kley 2010)

Vary Planet Mass $10 - 200M_{\text{Earth}}$

- Fixed $e_0 = 0.40$



- **e-damping for all planet masses.**

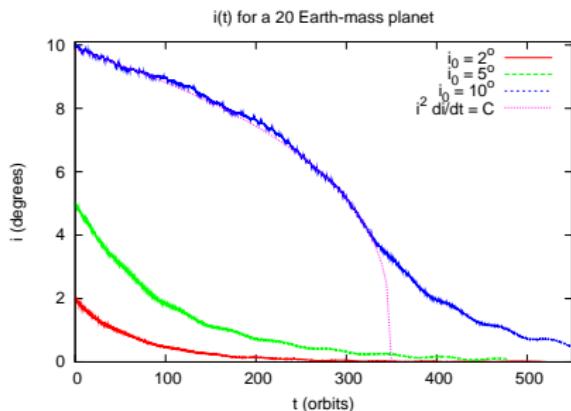
Small e : exponential damping, large e : $\dot{e} \propto e^{-2}$

- Damping timescale: $\approx (H/r)^2$ shorter than migration time

4.6 Elements: Inlined planets in 3D radiative disks

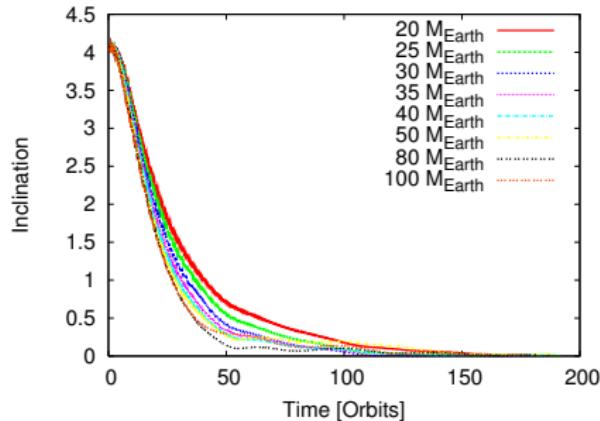
Fix planet mass $M_p = 20M_{\text{Earth}}$

- Vary initial Inclination



Vary Planet Mass $20 - 100M_{\text{Earth}}$

- Same $i_0 = 4\text{deg}$



(Cresswell ea 2007; Bitsch&Kley 2011)

- *i*-damping for all planet masses.
Small i : exponential damping, large i : $i \propto i^{-2}$
- Damping timescale: $\approx (H/r)^2$ shorter than migration time
Planets are confined to the disk midplane and are on circular orbits