Formation of massive planets

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Organisation

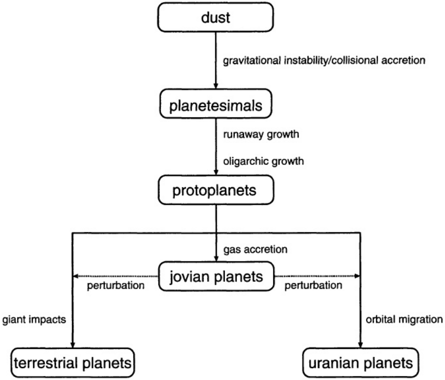
- **3.1 Concepts**
- **3.2 Core Accretion**
- **3.3 Final Mass**
- **3.4 Interior**

(National Geographics)

Scenarios for giant formation

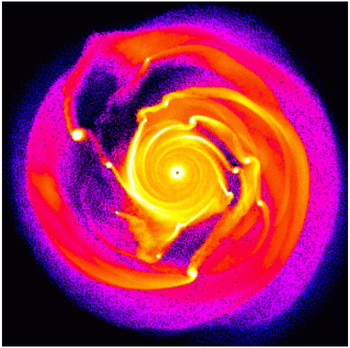
In principle 2 possibilities for giant planet formation

1) Core Accretion Model
or *core instability model*
first core formation
then gas accretion



(E.Kokubo)

2) Gravitational Instability model
or *disk instability model*
direct formation by grav. collapse
of the gas/dust disk
(global mechanism) $\Rightarrow$ Lecture 7

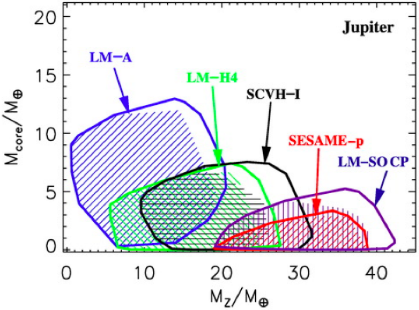


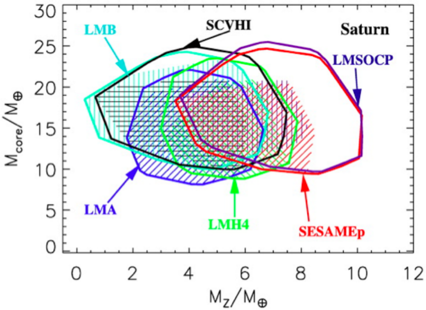
(L.Mayer)

From observations of mass, radius, gravitational moments, surface abundances

Jupiter and Saturn are enriched in heavy elements: Jupiter 1.5-6 solar, and Saturn 6-14 times solar. Interior structure modelling yield masses for the cores. ⇒ Core Accretion scenario fovoured for Solar System.

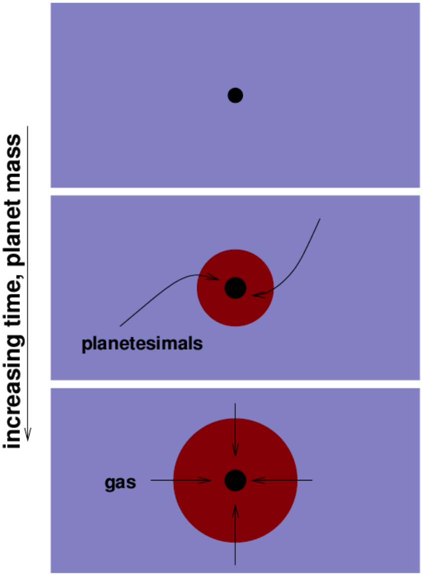
Fig.: Core mass vs. metal content for Jupiter & Saturn for different equations of state.





(Saumon & Guillot, 2004)

Principle core accretion



core formation
like terrestrial planets

Gas/Planetesimal accretion
hydrostatic phase
(thin) atmosphere forms

spherical gas accretion
(runaway process)

(P.Armitage)

Atmosphere accretion

When can a planet hold an atmosphere ?
(follow here (Armitage, 2010))

Weakest condition: $v_{esc} > c_s$ at the surface. Using

$$m_p = \frac{4\pi}{3} \rho_m R_p, \qquad v_{esc} = \sqrt{\frac{2Gm_p}{R_p}}, \quad \text{and} \quad c_s = \frac{H}{r} u_K$$

$\Rightarrow$ an icy body at 5 AU arund Solar mass star will have some atm for $m_p \gtrsim 5 \cdot 10^{-4} M_{earth}$. Very small mass but atm has no dynamical impact.

Consider situation where the envelope takes up a sizable fraction ϵ of the planets such that $M_{env} > \epsilon m_p$.
For an isothermal atm with $c_s(atm) = c_s(disk)$ and where the outer density matches that of the ambient disk (ρ_0) one obtains for:
$r = 5AU$, $\rho_0 = 2 \cdot 10^{-11}$g/cm^3, $c_s = 7 \cdot 10^4$cm/s and $\epsilon = 0.1$

$$m_p \gtrsim 0.2 M_\oplus \tag{1}$$

Role of the isolation mass

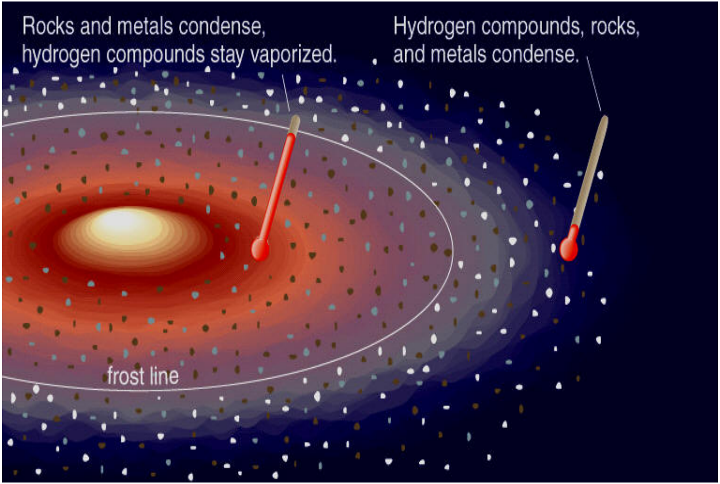
At the location of the Earth, one obtains for $\epsilon = 0.1$ with $r = 1AU$, $\rho_0 = 6 \cdot 10^{-10}$g/cm^3, $c_s = 1.5 \cdot 10^5$cm/s, and $\rho_m = 3$gm/cm^3

$$m_p \sim M_\oplus \tag{2}$$

- this is comparable to todays mass of the earth
- Planets are assembled over life time of the disk
- Important concept is the isolation mass, this must be larger than the mass to aquire an atmosphere
- The isolation mass is given by the amount of solids available at a given location in the disk
- This is given by the condensation temperature of a specific material
- The temperature in the disk drops with radius $\propto r^{-3/4}$ in active disk, or $\propto r^{-1/2}$ in passive disk

$\Rightarrow$ Check the condensation radius of water (snowline)

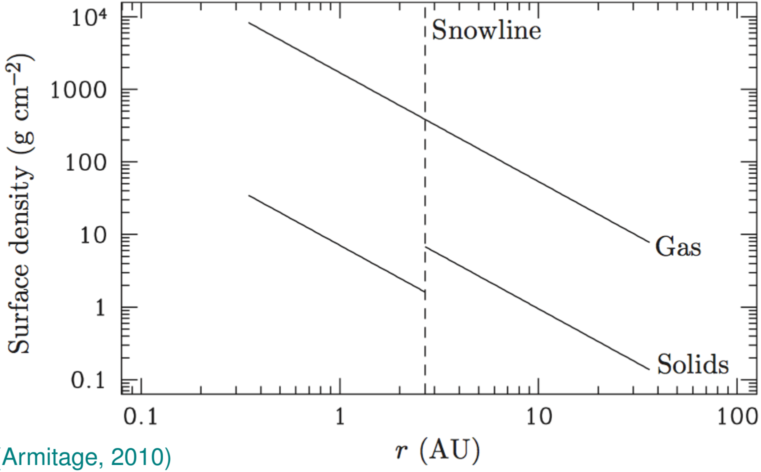
The snowline



(J.Baez from Pearson Education)

The location of the snowline depends on the evolutionary state of the disk, and whether it is turbulent or not (cf. dead zones), need $T \leq 150 - 170$ K

The snowline in the Solar System

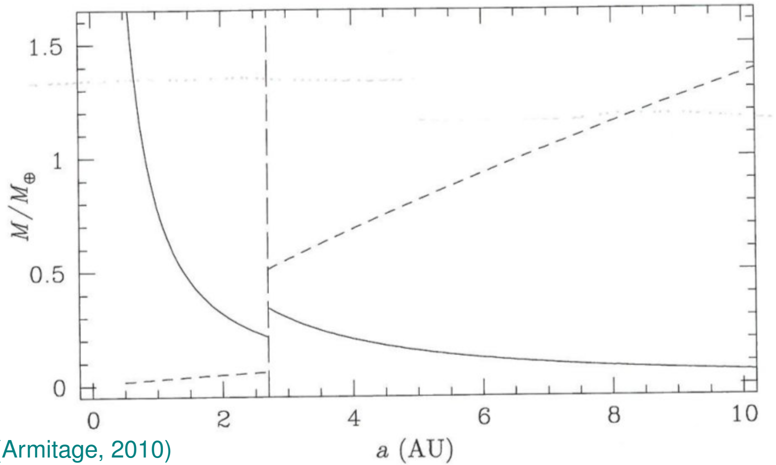


(Armitage, 2010)

$$\Sigma_s(rock) = 7.1\,(r/1AU)^{-3/2} \quad \text{g/cm}^2 \text{ for } \quad r < 2.7\text{AU} \tag{3}$$

$$\Sigma_s(rock/ice) = 30\,(r/1AU)^{-3/2} \quad \text{g/cm}^2 \text{ for } \quad r > 2.7\text{AU} \tag{4}$$

Snowline and isolation mass



(Armitage, 2010)

Dashed curve: isolation mass, solid curve: mass to sustain a massive envelope ($\epsilon = 0.1$).

Small radii: terrestrial type planets (no atm.), large radii: massive planets more likely, details depend on actual disk model

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(National Geographics)

Basic equations

Equations (hydrostatic), identical to stellar structure equations

mass conservation:
$$\frac{dm}{dr} = 4\pi r^2 \rho \tag{5}$$

Hydrostatics:
$$\frac{dp}{dr} = -\frac{Gm(r)}{r^2}\,\rho \tag{6}$$

radiative diffusion:
$$\frac{L(r)}{4\pi r^2} = -\frac{4\,\sigma T^3}{3\,\kappa_R \rho}\,\frac{dT}{dr} \tag{7}$$

energy generation:
$$\frac{dL}{dr} = -4\pi r^2 \rho \left(\epsilon - T\frac{\partial S}{\partial t}\right) \tag{8}$$

$m(r)$ Mass interior to the radius r; $L(r)$ luminosity at radius r;
ϵ: energy generation, σ Stefan-Boltzmann constant;
κ_R Rosseland-Opacity, S entropy (e.g. compressional heating).
In case of convection (super-adiabatic), take adiabatic T-gradient.

eos: ideal gas
$$p = \frac{R_{gas}\,\rho T}{\mu} \tag{9}$$

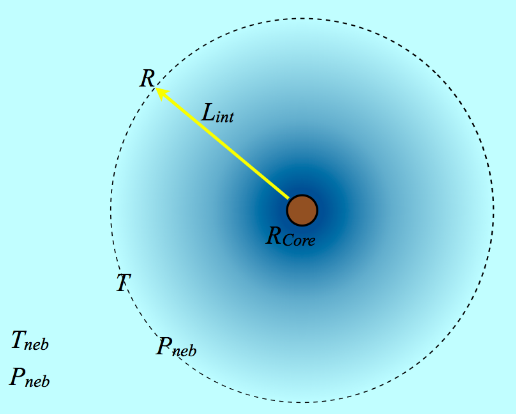
R_{gas} gas constant, μ mean molecular weight

Boundary conditions

Need boundary conditions to integrate structure equations at inner and outer radius

- Inner boundary (connected to size and luminosity of the core)
  R_{core}: from solution of structure equations for the core.
  First approximation: constant density for core
  L_{core}: from the residual kinetic energy of the accreted planetesimals
  Obtained from solving the infall equations.
  Additional core luminosity comes from:
  radioactive decay in the core,
  core contraction and cooling of the core.
- Outer boundary (atmosphere). Consider 3 different conditions
  1) attached (or nebular): The planet's envelope and the disk are continuous
  2) detached (or transition): The planet's envelope is no longer in contact with the disk
  3) isolated (or evolution): The planet is no longer in an accretion disk

Attached or nebular phase



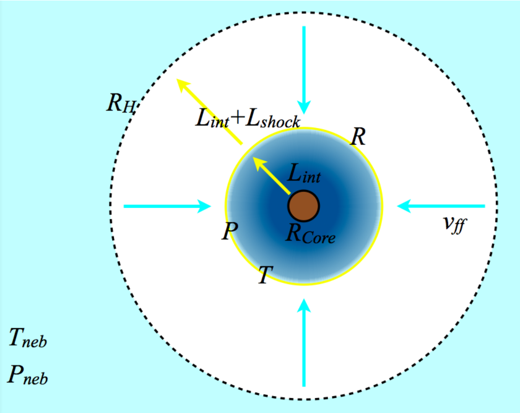
For small masses ($M_{core} <$ approx. $10 - 20 M_{earth}$) is the envelope of the protoplanet smoothly attached to the nebula. This fixes the outer temperature and pressure at the radius R.

(Chr. Mordasini)

R is given by the smaller of the accretion (Bondi) radius and the Hill radius, smoothly interpolated

$$R_A = \frac{GM_p}{c_s^2} \qquad \text{and} \qquad R_H = \left(\frac{M_p}{3M_*} \right)^{1/3} a \tag{10}$$

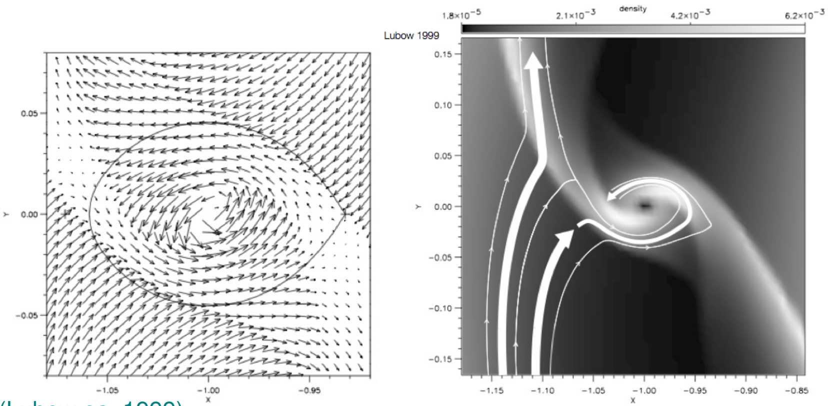
Detached or transition phase



For larger masses, no solution satisfying the attached state exists, and the proto-planet contracts to a radius much smaller than R_H.
The luminosity follows from radial infall (spherical accretion shock) or through accretion from a circumplanetary disk.

(Chr. Mordasini)

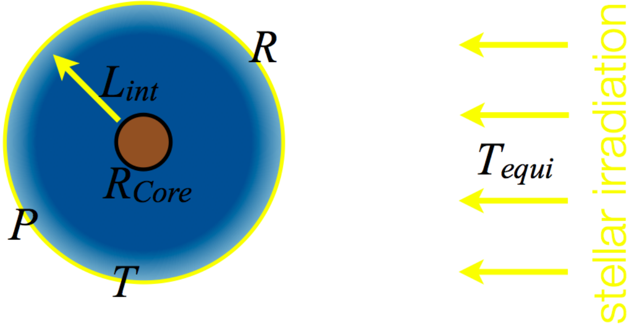
Gas accretion is not anymore determined by planet, but by nebula conditions
The pressure at the surface is determing by the accretion shock (gas in free-fall) and photospheric pressure



(Lubow ea, 1999)

2D planet-disk simulations for a 1 M_{Jup} planet in a viscous accretion disk
Left: velocity field around planet, the right/left $+$-signs are the L_1/L_2-points
Right: flow field (in co-rotating rame), the matter in the left orbits the star, returns and can be accreted
The simulations show that the planet accretes material via 2 'streams'
$\Rightarrow$ corrections to previous spherically symmetric picture

Isolated or evolution phase

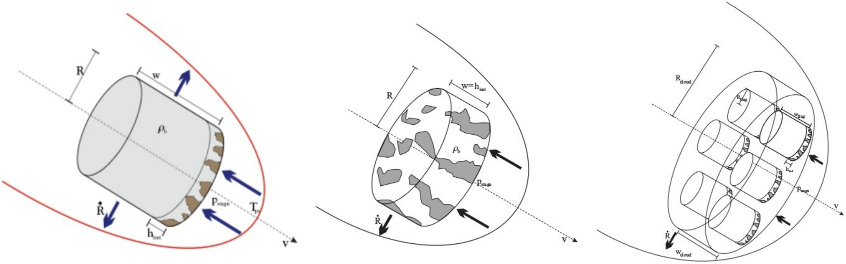


(Chr. Mordasini)

planet evolves at constant mass
receives irradiation from central star, spread over planetary surface
need planet albedo, photosphere at $\tau = 2/3$

Luminosity is provided by the accretion of planetesimals (methods from previous lecture)
Planetesimal-envelope interaction is important for the energy and mass deposition profile in the envelope. Have to consider the following points:

- gravity and gas drag (envelope increases capture radius)
- thermal ablation and aerodynamical disruption (cf. Showmaker-Levy 9) reach over 30,000 K in bow shock around infalling body (SL9 type) heating of planetesimal ⇒ re-radiation, heating, mass loss,



Mizuno (1980):
Core-accretion scenario (core-instability)

- hydrostatic equations
- all planetesimals reach the core $\Rightarrow$ luminosity L with $L(r) = const.$
- he showed that there is a critical core mass, M_{crit}, above which there is no hydrostatic solution for the envelope possible.
- M_{crit} is a function of the opacity of the gas
- is around $10 M_{earth}$ for ISM materials.

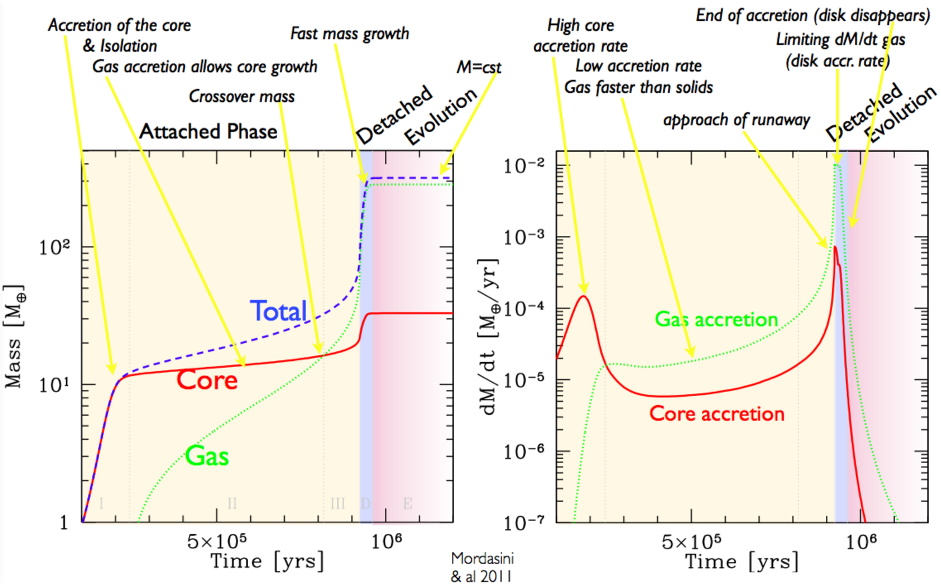
There is a simple analytic model (Stevenson, 1982), see (Armitage, 2010)

For typical parameter (at 5AU)

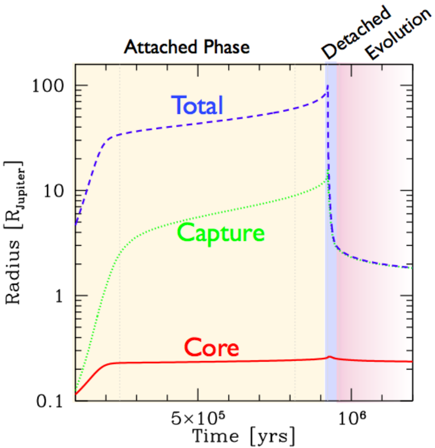
$$M_{crit} \simeq 20 \kappa_R^{3/7} \, M_{\oplus} \tag{11}$$

with κ_R in units of [cm^2/g]

Above $M_{core} = M_{crit} \Rightarrow$ Contraction and Runaway Accretion
Need numerical simlations to obtain detailed evolution

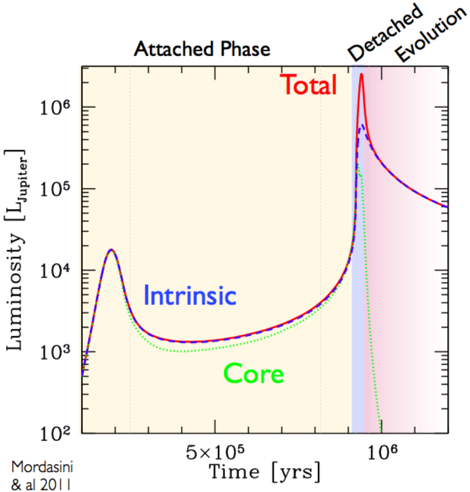


Mordasini & al 2011



The plot shows the total/outer radius, the capture radius and the core radius as a function of time.

Mordasini & al 2011



Mordasini & al 2011

$$L_{Jup} = 8.7 \times 10^{-10} L_{\odot}$$

Summary

Qualitative evolution
$\approx$ as in plot on right
typical growth time a few 10^6 yrs

But details are not known:

Opacity: κ_R
lower values in envelope allow faster growth, depends on amount of dust

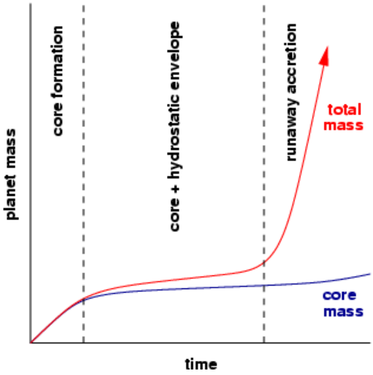
Convection: in envelope
changes radiation transport

Chemical composition: μ

Accretion rate: $\dot{M}_{env}$
one dimensional vs. multi-dimensional models

Migration through disk
radial migration changes accretion rate

Schematic Growth



(Scholarpedia)

Core formation time $< 10^6$ yrs
Hydrostat. phase: some 10^6 yrs
Mass at this point $\approx 10 M_\oplus$
Start Runaway at $(5-10) \times 10^6$ yrs

Discussion

Advantage:
explanation of the cores of Solar System planets

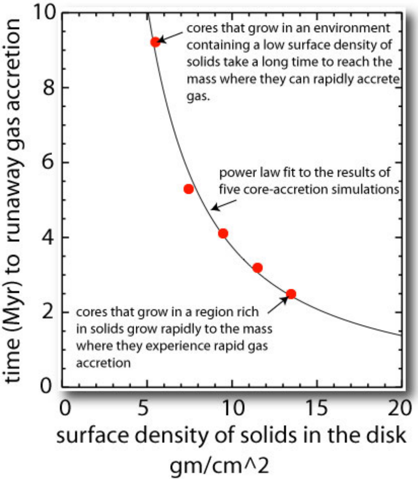
But:
Parameter fine tuning necessary

For $\Sigma_{\mathrm{solid}} > 10\mathrm{g/cm}^2$: too many heavy elements in comparison to todays Jupiter

For $\Sigma_{\mathrm{solid}} < 10\mathrm{g/cm}^2$: core formation too long, no gas left over (more neptune mass planets around low mass stars ?)

Possible ways out:
- pebble accretion
- rearrangment of planets (Nice model $\Rightarrow$ lecture 8)

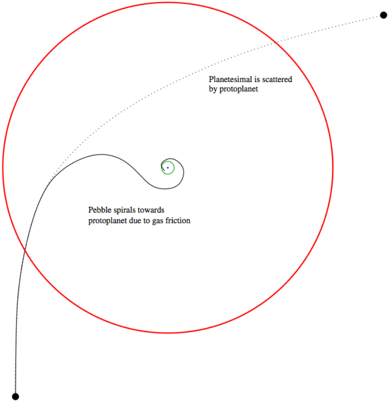
Timescale problem



(Image: http://oklo.org/)

Pebble accretion I

Idea: Enhance growth of planetary cores by rapid accretion of small pebbles (dm-sized) (Lambrechts & Johansen)

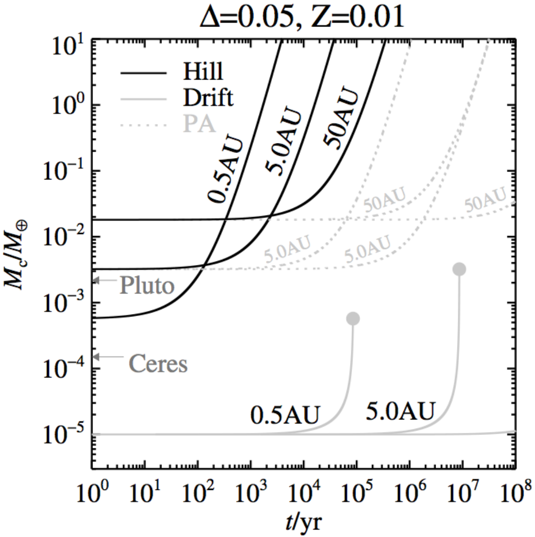


blue: planetary core
green: gravitational radius (cross-section)
red: Hillsphere
dashed: path of planetesimal
solid black: path of pebble

For pebbles the effective capture radius is much larger than that for planetesimals.
Because: Pebbles feel the gas drag that slows them down
Spiral in towards planet
Define: $\alpha_p \equiv R_p/R_H \ll 1$

Planetesimal accretion: $\dot{M}_p = \alpha_p R_H^2 F_{grav}$
Pebble accretion: $\dot{M}_p = R_H^2 F_{grav}$

Pebble accretion II



Assuming a steady inflow of material (pebbles)
Then fast mass growth can be achieved within lifetime of disk

Up to 10 $M_\oplus$ at 50 AU within 1 mio. yrs
shorter times: 2D; longer: 3D

Headwind-parameter $\Delta = \Delta u_\phi / c_s$
Vertical pebble concentration: $Z = H_p / H$

Details: to be worked out !

(Lambrechts & Johanson, 2012)

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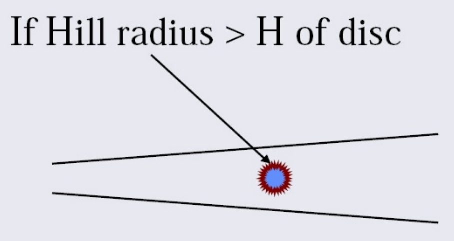
(National Geographics)

In the formation region of Jupiter would (probably) have been enough gas for a much larger planet. What limits the growth?

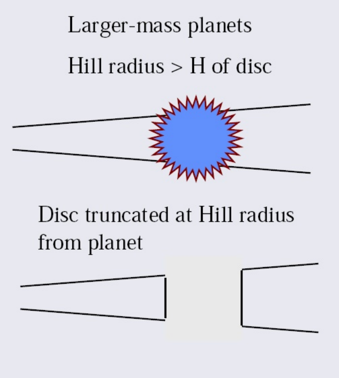
Low mass planet:
$R_{Hill} < H$(disk)



(P. Armitage)

Disk not influenced
$\Rightarrow$ little influence on growth

Large mass planet:
$R_{Hill} \geq H$(disk)



(P. Armitage)

disk interrupted
$\Rightarrow$ influence on growth

Gap formation I

What creates a gap in the disk at the location of the planet ?

- Only partially the accretion of material onto the planet
- Mainly angular momentum transfer from the planet onto the disk

Two point of views:

- Impulse approximation (consider motion of individual particles)
- Hydrodynamical (superposition of sound waves)

Width of the gap is determined by gravity, viscosity and gas pressure
Investigate first angular momentum (a.m.) transport due to gravitational interaction of particles with planet

Gap formation II

Impulse approximation: Particles in the gas disk are only perturbed in the direct neighbourhood of the planet (●). deflection about angle δ for passage (Kepler: hyperbolic orbit)

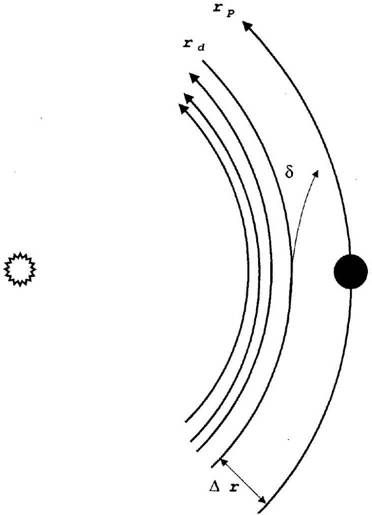
$$\cot^2(\delta/2) = v_{\rm rel}^2 \Delta r^2/(G^2 m_p)$$

relative velocity $v_{\rm rel} = r_d \Omega(r_d) - r_p \Omega_p$.
$\Rightarrow$ specific a.m. change of the gas per passage (about Δv_φ)

$$\Delta j = -v_{\rm rel} r_d (1 - \cos\delta) \approx -\frac{2 G^2 m_p^2 r_d}{(\Delta r)^2 v_{\rm rel}^3}$$

Passage: all $\Delta t = 2\pi/|\Omega - \Omega_p|$
i.e. a.m. exchange rate: $\dot{j} = \Delta j/\Delta t$
Total exchange

$$\dot{J} \equiv \int_{\Delta r_0}^{\infty} \Sigma \dot{j}\, 2\pi r d(\Delta r) \qquad (12)$$



(Cassen, 2003)
From: *Saas-Fee Advanced Course 31: April 2001*

Gap formation III

Expand $\Omega(r)$ around Ω_p and integrate

$$\dot{J}_{grav} = -\frac{8}{27}\left(\frac{r_p}{\Delta r_0}\right)^3 \left(\frac{m_p}{M_*}\right)^2 \Omega_p^2 \Sigma r_p^4 \tag{13}$$

Notes:

- in spite of simple approximation is result nearly exact.
- in reality $\dot{J}$ follows from complex wave phenomena
- minus sign: Inner disk loses a.m., while outer gains a.m.
- in vicinity of planet ($\Delta r_0 \to 0$) strong increase of $\dot{J}$

$\Longrightarrow$ in the planet region the disk will be 'pushed' away (gap)

For the viscous torques ($\dot{J}$) at the location of the planet

$$\dot{J}_{visc} = \dot{M}_{disk} j_p = 3\pi\Sigma\nu\, r_p^2 \Omega \tag{14}$$

for gap formation $\dot{J}_{grav} \geq \dot{J}_{visc}$. Let smallest $\Delta r = R_{\rm Hill}$, then

$$q \geq q_{visc} \simeq \frac{10\nu}{\Omega_p\, r_p^2} \qquad (\text{with} \quad q = \frac{m_p}{M_*}) \tag{15}$$

Gap formation IV

A second criterion of gap formation is obtained from the condition that the Hill sphere of the growing planet is larger than the disk thickness, $R_{\rm Hill} \geq H$

$$q \geq q_{Hill} \simeq 3 \left(\frac{H}{r} \right)_p^3 \tag{16}$$

For typical parameter of the protoplanetary disk both criteria yield similarly the a limiting mass for gap formation: between Jupiter and Saturn mass.

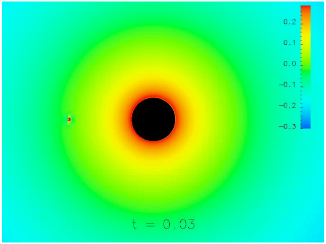
This explained the mass of Jupiter quite well, i.e. good agreement with Solar System
subsequent hydrodynamical simulations showed that the mass can increase further
i.e. good agreement with extrasolar planets

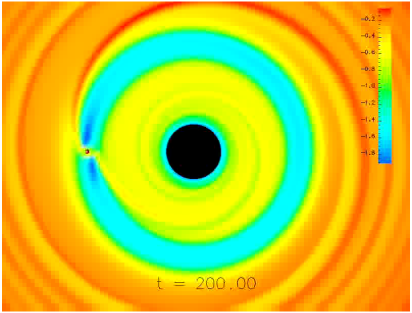
($\rightarrow$ Lecture 6)

Dynamical gap formation

M_p = 1 M_{Jup}, a_p = 5.2 AU, and disk around 1 $M_\odot$ star

hydrodynamical evolution (solve Navier-Stokes equations)





General criterion for gap formation (Crida et al., 2006)

$$\frac{3}{4}\frac{H}{R_{Hill}} + \frac{50}{q\,Re} \leq 1 \tag{17}$$

with $q = m_p/M_*$, $Re = a_p^2\Omega_p/\nu$.

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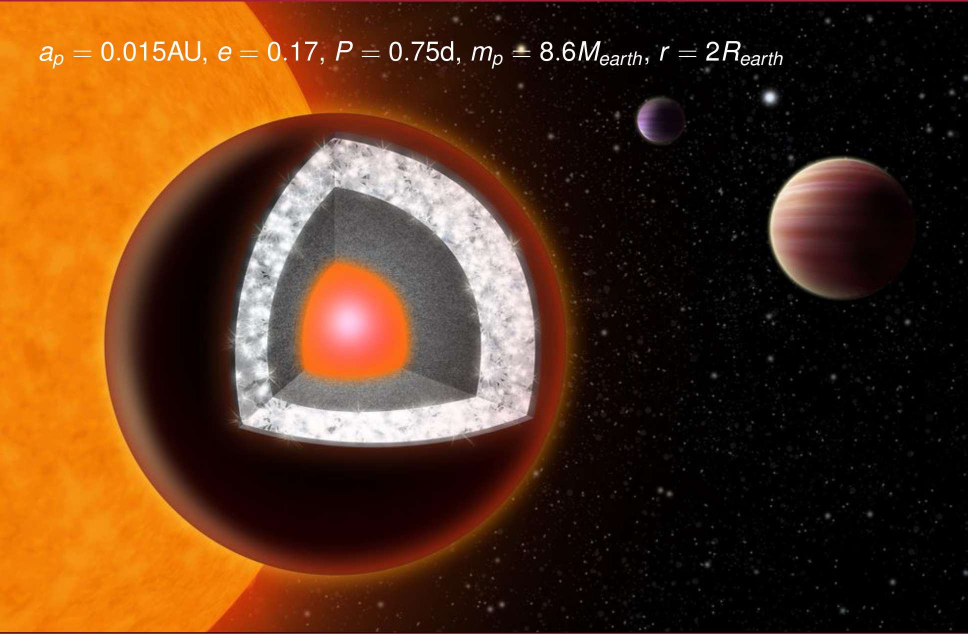
(National Geographics)

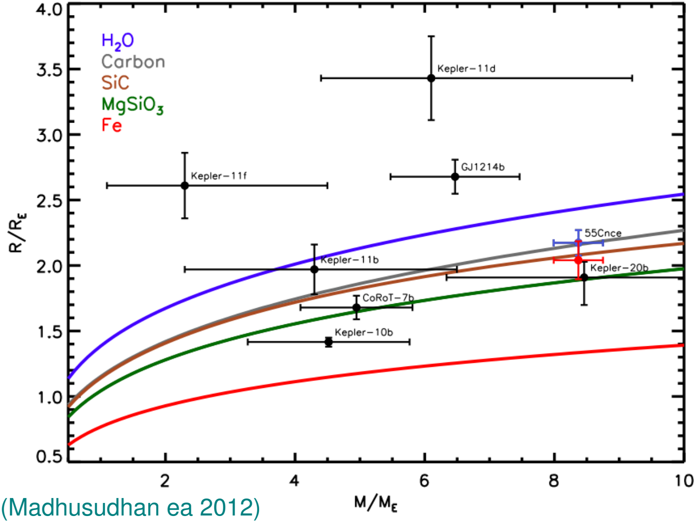
The Diamond planet: 55 Cnc e

$a_p = 0.015\text{AU}$, $e = 0.17$, $P = 0.75\text{d}$, $m_p = 8.6 M_{earth}$, $r = 2 R_{earth}$



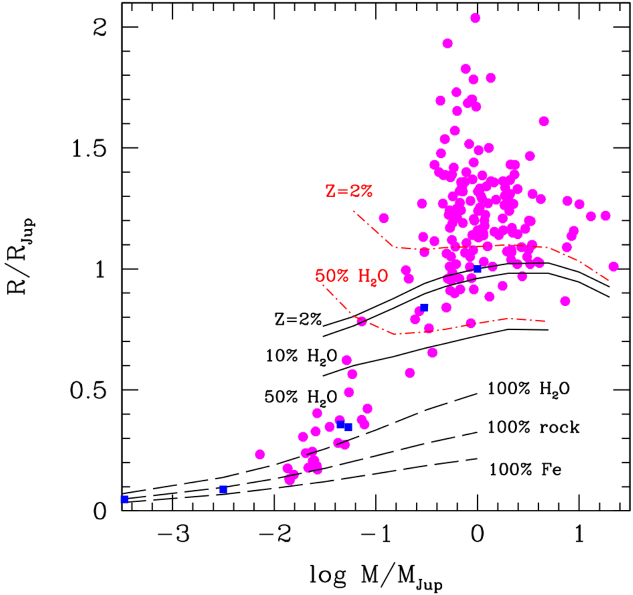
Interior structure

Mass-radius relation for a number of planets compared to homogeneous models. The exoplanet 55 Cnc lies exactly on the 100% carbon curve.



(Madhusudhan ea 2012)

Mass-Radius relation - Observation



Observations:
Solar Planets
- Mars to Neptune

Exoplanets
- cleaned sample

Models:
dashed lines:
- pure systems

solid lines:
- mixtures

in red:
- irradiated planets

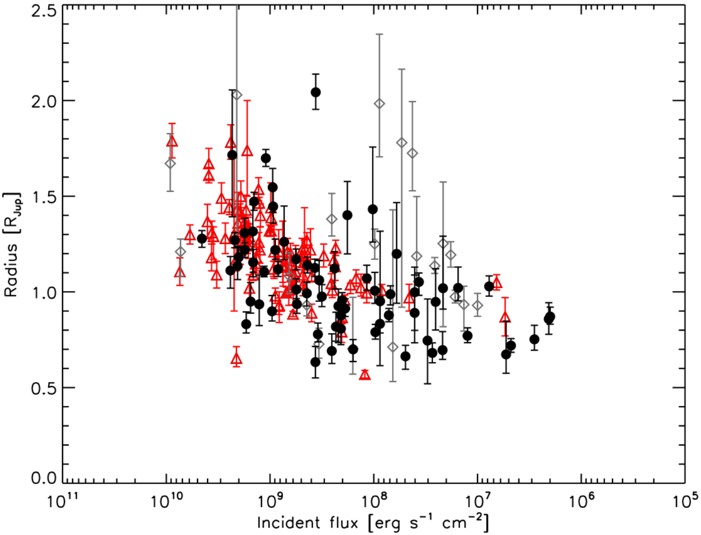
(Baraffe, 2014)

Mass-Radius relation - Theory

Explaining the M-R relation

- Need internal structural model
  - hydrostatics
  - EOS: equation of state (up to 20,000 K and 70 Mbar)
  - from experiments (eg. shock-wave) and theory (QMD)
  - for Hydrogen and heavy elements, phase transitions
- Earth-like planets (thin atmosphere)
  - cooling through convection (very effective)
  - plate tectonics (viscosity)
  - heating sources (radioactivity, remnant heat)
  - about a dozen exoplanets with $R < 2R_\oplus$
- Giant Planets (extended atmosphere)
  - irradiation by star, evaporation of atm.
  - inflated exoplanets (stellar flux, tidal effects, delayed contraction)

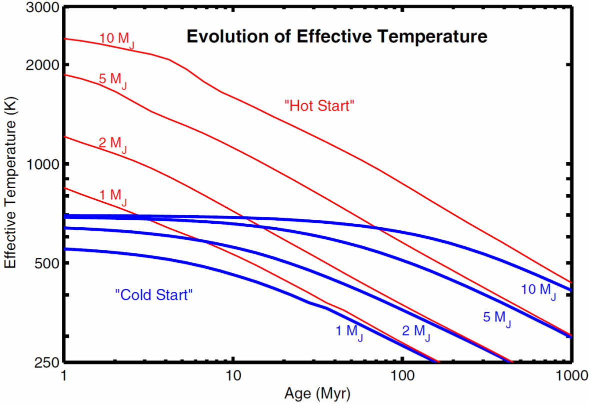
Stellar Flux - Planet Radius (Relationship)



Need for global 3D circulation models (Demory & Seager, 2011)

Hot vs. Cold start

Evolution of effective temperature for giant planets with 1, 2, 5 and 10 times Jupiter's mass.
Cold start refers to core accretion models (gas in disk can cool efficiently), while disk instability leads to rapid collapse where the planet remains hot for some extended time.



(Spiegel & Burrows