Growth of terrestrial planets

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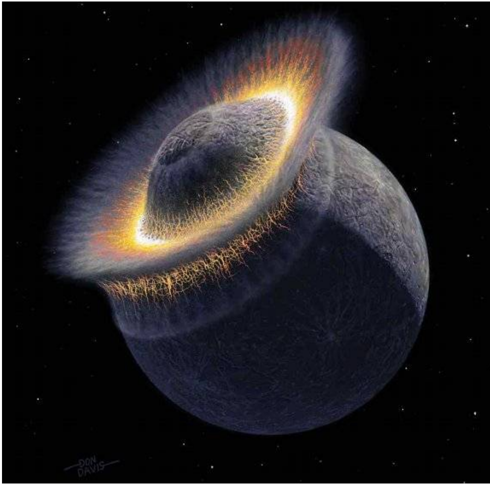
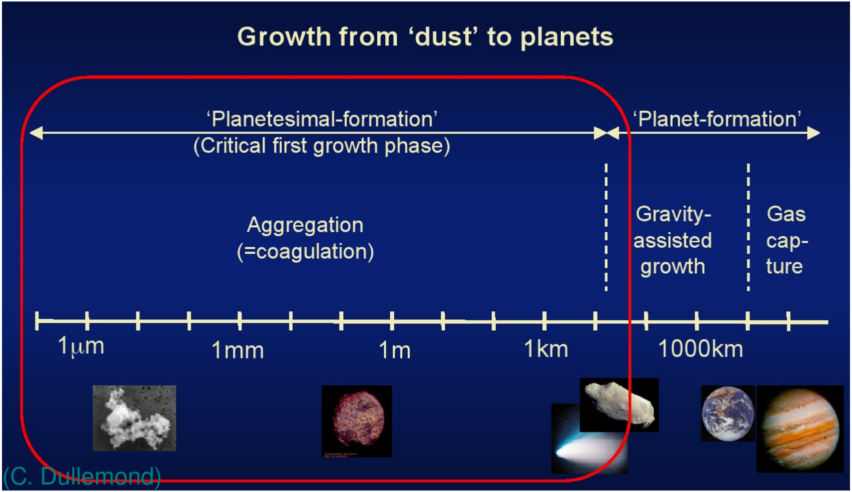
Lecture overview:

- **2.1 Concepts**
- **2.2 Protoplanets**
- **2.3 Terrestrial Planets**
- **2.4 Water**
- **2.5 Moon formation**



(Credit: Don Davis)



Dust ⇒ Planetesimals (μm ⇒ 1-10km, direct collisions)
Concentration of particles (eddies, vortices, pressure bumps),
timescale $\approx 10^5$ yrs.

2.1 Concepts: The problem

Planetesimals:
- Objects from 1-10km up to nearly moon-sized (planetary embryos)
- Starting point for later phase of planet formation
- Now gravitational interaction becomes important

In this mass range: very small aerodynamic drag forces
Possible: Inhomogeneities of the disk density
$\rightarrow$ tidal interaction

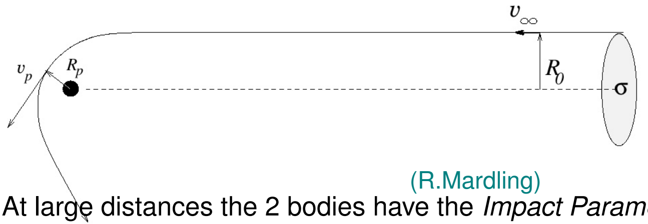
With initial 1 km-sized particles need 10^{11} particles to make the terrestrial panets.

Numerically very demanding:
- a lot of particles
- very long evolution timescale (many dynamical times)

$\Longrightarrow$ Combination of statistical and numerical methods

Gravitational focussing I

Two bodies can only grow via physical collisions
Mutual gravitational interaction increases the effective crosssection



(R.Mardling)

At large distances the 2 bodies have the *Impact Parameter* R_0 and velocity $v_{\rm rel} = v_\infty$, the shortest distance is R_p with velocity v_p.
Angular momentum conservation

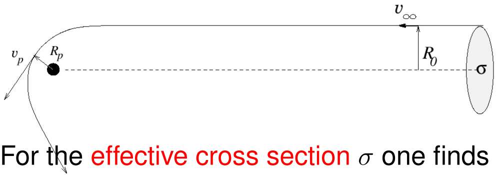
$$R_0\, v_{\rm rel} = R_p\, v_p \tag{1}$$

energy conservation

$$\frac{1}{2}\mu\, v_{\rm rel}^2 = \frac{1}{2}\mu\, v_p^2 - \frac{G(m_1 m_2)}{R_p} \tag{2}$$

with the reduced mass $\mu = m_1 m_2/(m_1 + m_2)$.

Gravitational focussing II



(R.Mardling)

For the effective cross section σ one finds

$$\sigma \equiv \pi R_0^2 = \pi R_p^2 \, F_{\rm grav} = \pi R_p^2 \left[1 + \left(\frac{v_{\rm esc}}{v_{\rm rel}} \right)^2 \right] \qquad (3)$$

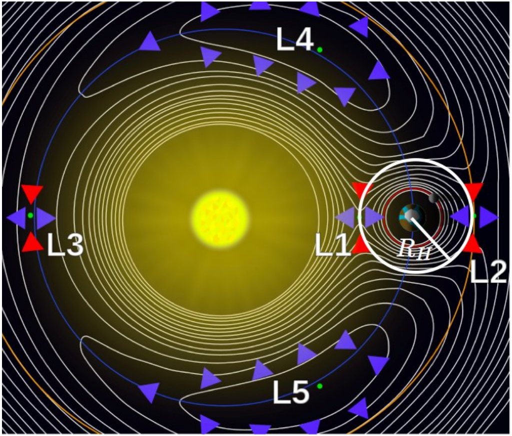
with the gravitative enhancement factor $F_{\rm grav}$ and the escape velocity

$$v_{\rm esc} = \left(\frac{2G(m_1 + m_2)}{R_p} \right)^{1/2} \qquad (4)$$

In a cold disk of planetesimals with $v_{\rm rel} \ll v_{\rm esc}$ the cross section is MUCH higher than without gravity.
Notes: Safronov number $\theta = (v_{esc}/v_{rel})^2$. For 2 bodies with sizes r_1 and r_2 set $R_p \rightarrow r_1 + r_2$.

Hill sphere

Equipotential lines in the co-rotating frame

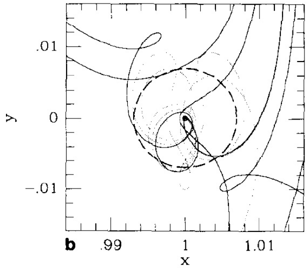


The Hill sphere (white line) is the region where the gravity of the planet (here growing planetesimal) is dominant.
It is enclosed within the two Lagrange points L_1 and L_2.
The Hill-Radius R_{H}, is given by

$$R_{\mathrm{H}} = \left(\frac{m_p}{3M_*}\right)^{1/3} a_p \qquad (5)$$

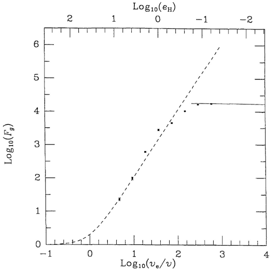
where a_p is the semi-major axis of the planet.

Three-body effects



Trajectories in 3-body problem
very complex (chaotic), in particular in Hill sphere (dashed circle)
(here: Star and two planetesimals)

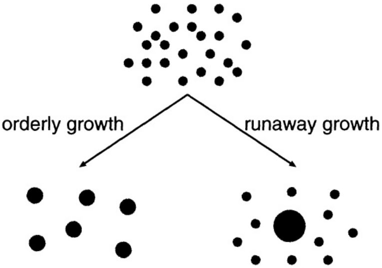
(Greenzweig & Lissauer, 1993)



Gravitational focussing factor F_{grav}
as a function of $v_{\mathrm{esc}}/v_{\mathrm{rel}}$

dashed: Eq. (3)
Note: $v_{\mathrm{esc}}/v_{\mathrm{rel}} \gg 1$ means very thin disk:
3-body effects limit F_{grav} (solid line)
to values up to about 10^4.
(Lissauer, 1993)

Modes of Growth



Two possible modes:

Ordered

Mass ratio of two growing particles tends to unity

Runaway

Large particles grow faster than small ones

(Kokubo, 2001)

Consider growth of two particles with mass m_1 and m_2 with $m_1 > m_2$

$$\frac{d}{dt}\left(\frac{m_1}{m_2}\right) = \frac{m_1}{m_2}\left(\frac{1}{m_1}\frac{dm_1}{dt} - \frac{1}{m_2}\frac{dm_2}{dt}\right) \tag{6}$$

i.e. relative growth $1/m(dm/dt)$ is important.

If relative growth increases with m: Runaway-Growth
If relative growth decreases with m: ordered groth
Look at mass growth

Using the cross section σ (Eq. 3) the mass growth of a planetesimal with mass m_p is given by

$$\dot{m}_p = \rho_{\rm part}\, v_{\rm rel}\, \sigma = \rho_{\rm part}\, v_{\rm rel}\, \pi R_p^2\, F_{\rm grav} \tag{7}$$

if each collision will results in growth (100% *sticking*).
$\rho_{\rm part}$ = density of incoming particles. Using

$$\rho_{\rm part} \approx \frac{\Sigma_{\rm part}}{2H_{\rm part}} = \frac{\Sigma_{\rm part}\Omega_K}{2v_{\rm rel}} \tag{8}$$

with $H_{\rm part} \sim v_{\rm rel}/\Omega_K$ and $v_{\rm rel} \approx \sqrt{e^2+i^2}\, v_K$ (use here the velocity dispersion of the planetesimal disk)

$$\frac{dm_p}{dt} = \frac{1}{2}\Sigma_{\rm part}\Omega_K \pi R_p^2 \left[1 + \left(\frac{v_{\rm esc}}{v_{\rm rel}}\right)^2\right] \tag{9}$$

- Growth proportional to $\Sigma_{\rm part}$
- Growth proportional to Ω_K: i.e. slower at larger distances
- $v_{\rm rel}$ enters only through focussing factor

Note: With increasing mass the growing planet influences the velocity dispersion ($v_{\rm rel}$) and the surface density $\Sigma_{\rm part}$.

show two illustrative cases:

- Ordered
  With $F_{\text{grav}} = const.$, and denote $m = m_p$ we have

$$\frac{1}{m}\frac{dm}{dt} \propto m^{-1/3} \tag{10}$$

  This implies a linear growth with radius: $R_p \propto t$

- Runaway
  Take now $v_{\text{rel}} = const.$ then

$$\frac{1}{m}\frac{dm}{dt} \propto R_p \propto m^{1/3} \tag{11}$$

  This implies $m \to \infty$ in a finite time!

Upon mass growth of a growing body velocity and density of the ambient planetesimals will be changed $\Rightarrow$ Modifications

Look now at the growth in more detail: Results from numerical simulations

Organisation

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(Universe Today)

Direct N-Body

solve equation of motion for N planetesimals

$$\frac{d\vec{v}_i}{dt} = -GM_\odot \frac{\vec{x}_i}{|\vec{x}_i|^3} - \sum_{j\neq i}^{N} Gm_j \frac{\vec{x}_i - \vec{x}_j}{|\vec{x}_i - \vec{x}_j|^3} + \vec{f}_{gas} + \vec{f}_{col} \tag{12}$$

$\vec{f}_{gas}$: friction force by gas particles
$\vec{f}_{col}$: veloc. change upon collisions

The velocity dispersion of the particles v_{disp} is damped by these forces.

Advantage: acurate methode
Disadvantage: need very many particles

Statistical:

solve for probalility distribution function $f(\vec{r}, \vec{v})$, expressed thru $f(e, i)$
particle density $n = \int f d^3 v$
Solve: a) Boltzmann-equation

$$\frac{\partial f}{\partial t} + \dot{\vec{r}} \frac{\partial f}{\partial \vec{r}} + \dot{\vec{v}} \frac{\partial f}{\partial \vec{v}} = \left. \frac{\partial f}{\partial t} \right|_{\text{coll}} + \left. \frac{\partial f}{\partial t} \right|_{\text{grav}} \tag{13}$$

coll: changes by collisions
grav: grav. scattering

and b) Coagulation equation

$$\frac{dn_k}{dt} = \frac{1}{2} \sum_{i+j=k} A_{ij} n_i n_j - n_k \sum_{i=1}^{\infty} A_{ik} n_i \tag{14}$$

with $n_k \propto$ number of particles with a give size
Advantage: model total ensemble
Disadvantage: only statistical

Runaway Growth I

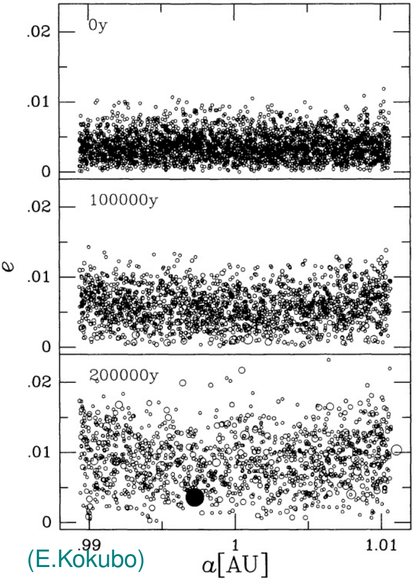
Example N-body - simulation

Planetesimals in ring at 1AU
with width $\Delta a = 0.02$AU
3000 bodies with each $m = 10^{23}$ g
with density $\rho = 2$gcm^{-3}

at time $t = 200,000$yrs:
- 1 body (●) with 100 initial masses

low eccentricity of ●:
- through dynamical friction
- small bodies have higher e
- large have smaller e

In early phase growth is through a Runaway phase



(E.Kokubo)

Same N-body - simulation:

dashed: 10^5yrs, solid: 2×10^5yrs

objects between 10^{23}-10^{24}g contain majority of mass

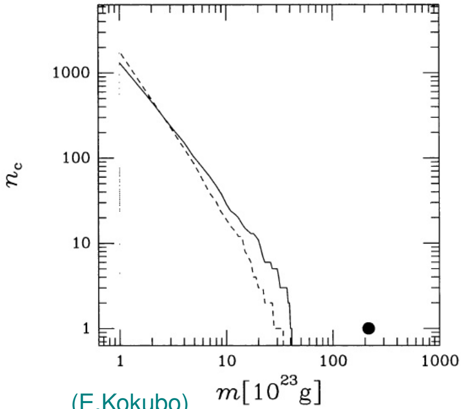
cumulative mass distribution follows: powerlaw

$$\frac{dn_c}{dm} \propto m^\alpha \qquad (15)$$

Here $\alpha \simeq -2.5$
($\alpha < -2.0$ is characteristical for runaway)

one very massive particle (●) separated from distribution (sink)

Cumulative mass distribution
$n_c(m)$ = number of particles with mass $> m$



(E.Kokubo)

Gravitational Stirring

Gravitational interaction between small and big bodies

Increases mean eccentricity and inclination of small bodies

equipartition of energy between e and i gives $< e^2 >= 4 < i^2 >$

$< x >$: mean values

solid line e
dashed line i

(E.Kokubo)

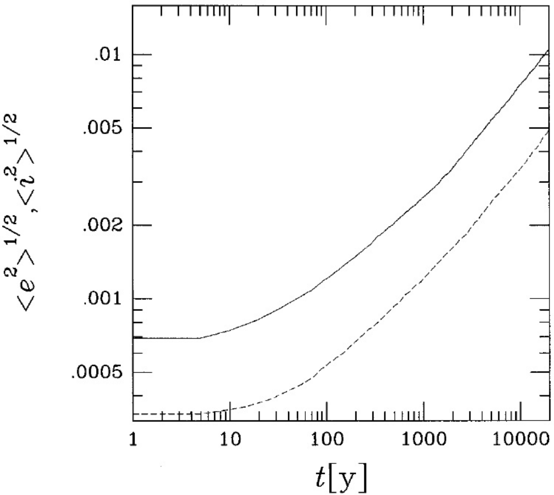
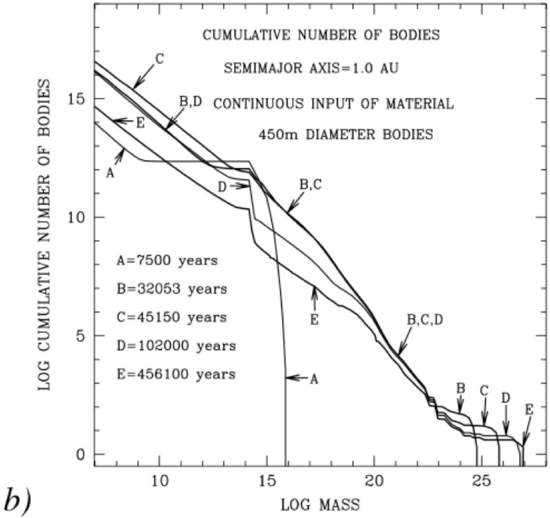


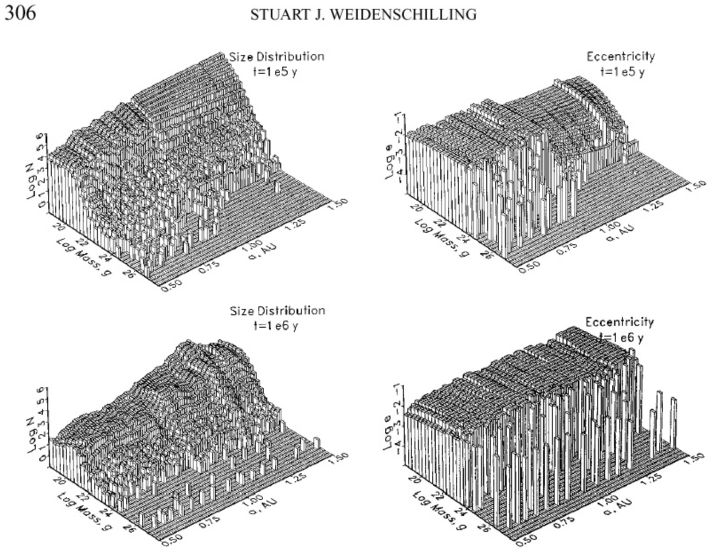
FIG. 6. The RMS eccentricity (solid curve) and inclination (dashed curve) as a function of time.

Example: Statistical simulation in box at 1AU, $\Delta a = .17$AU



(Wetherill & Stewart, 1993)

Example: Statistical simulation, 100 radial zones, for $m > 10^{24}$ discrete



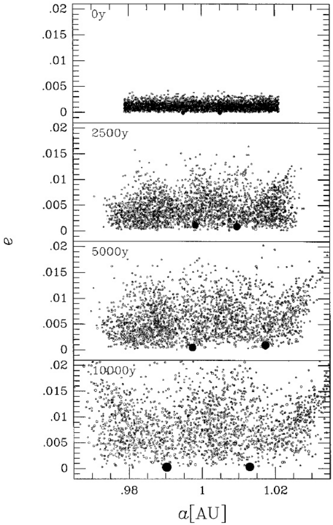
Simulation results: a few massive embryos with equal spearation

since N small, N-body is now more efficient
continue the above N-body simulation

4000 bodies, each $m = 1.5 \times 10^{23}$ g
add 2 seed-protoplanets: $M_1 = M_2 = 40m$
at $t = 0$ in $\Delta a = 0.042$AU
4 times larger radii ($f = 4$)
i.e. larger time scales

Result:
- large bodies grow at same speed
  $M_{end} \approx 8 M_{init}$
- small grow slower, $\bar{m}(t = 10^4) \approx 1.6 m_{init}$
- large have lower e

Self-limited runaway
for larger M a growth of v_{disp}
above $M \approx 50m$: $v_{disp} \propto M^{1/3}$
the $(1/M) dM/dt \propto \Sigma_p M^{-1/3}$
$\Rightarrow$ ordered growth



(Kokubo&Ida, 1998)

2.2 Protoplanets: End of growth - Isolation mass

Runaway is local

large bodies have $\approx$ circular orbit $\Longrightarrow$ limited reservoir of collision partners $\Longrightarrow$ Isolation Mass (M_{iso})

Collisions occur with bodies from *Feeding Zone*, i.e. from a region with extension of the Hill-radius

$$R_{Hill} = a \left(\frac{m_p}{3M_\odot} \right)^{1/3}$$

Particles come from region Δa with mass $m = 2\pi\, 2a\, \Delta a \Sigma_p$, let $\Delta a = C R_{Hill}$ with $m = M_{iso}$ we obtain

$$M_{iso} = 4\pi a C \left(\frac{M_{iso}}{3M_\odot} \right)^{1/3} a\Sigma_p \qquad (16)$$

detailed simulations result in $C = 2/\sqrt{3}$

End of growth - Examples

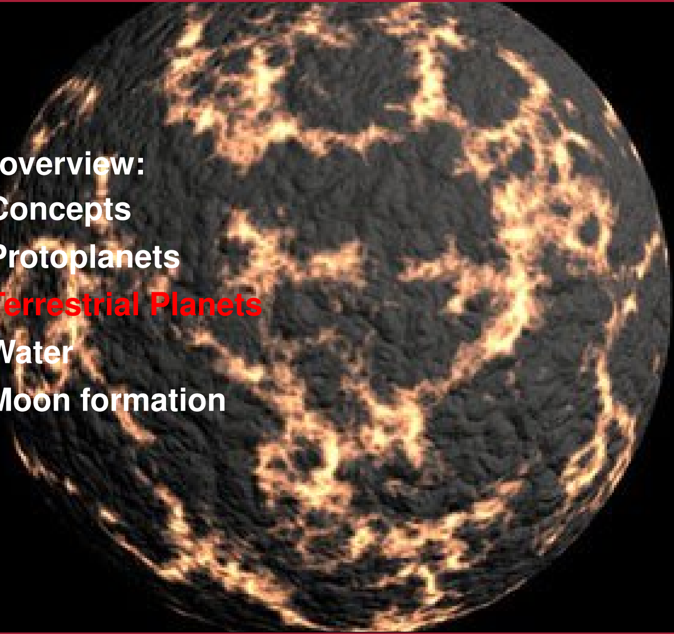
Let $2M_\oplus$ between 0.5 und 1.5AU, $\Sigma_p \sim a^{-3/2}$ and $\Sigma_p = 8$ gcm^{-3} at 1AU and $C = 2/\sqrt{3}$, then

$$M_{iso} \approx 0.05 M_\oplus \tag{17}$$

I.e. about 40 bodies (Proto-planets) with a mean distance of $\Delta a \approx 0.025$AU

At the distance of Jupiter one gets

$$M_{iso} \approx 5 - 9 M_\oplus$$



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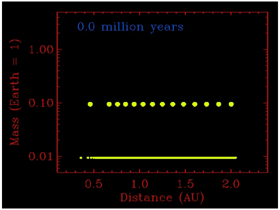
From moons to Earth

After oligarchic phase: only few objects of $\approx$ moon size left over
Only gravitational interaction between these embyos

Approach: classical N-Body simulations

New difficulty:
few particles ($\approx$ 100), but very long timescales (10^8 yrs)
$\Rightarrow$ need good (symplectic) Integrators

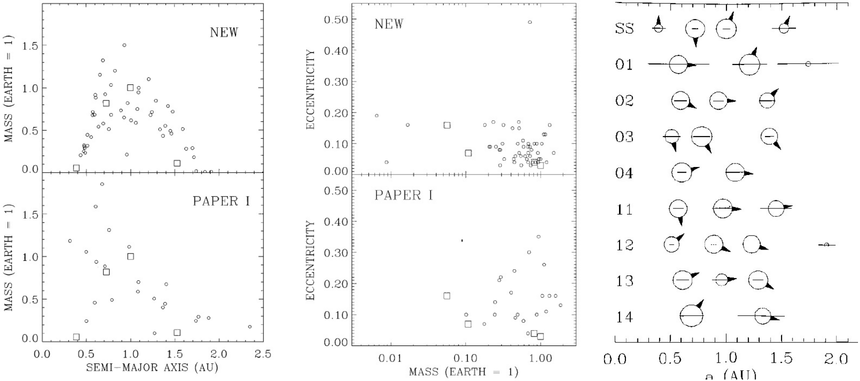
Example: (Chambers, 2001)
16 N-body simulations, Start with 153-158 embryos
distribute ca. 2 Earth masses between 0.3 and 2.0 AU
different types: all masses equal, bimodal, radial mass profile
include Jupiter & Saturn (on present orbits)
100% sticking (perfectly inelastic)
angular momentum into rotation (spin)



Integrator:
(`Mercury`-Package, John Chambers, 1999)

Systems similar to the Solar System are possible
mostly 3-4 terrestrial planets, formation in ca. 10^8 yrs

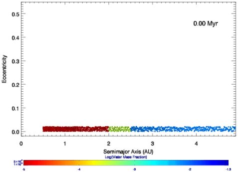
But: Discrepancies in important details
- often no high mass concentration as in Venus and Earth
- planets have too large *e* and *i* compared to Solar System
- spin orientientations arbitrary

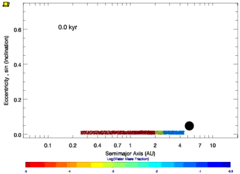


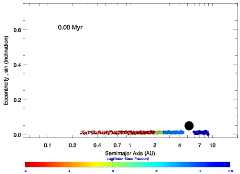
(Chambers, 2001); Paper I (with less particles): (Wetherill & Chambers, 1998)

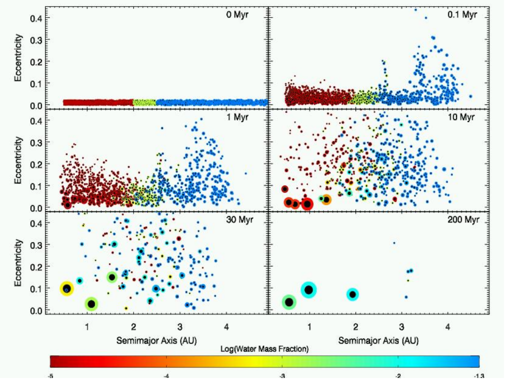
2.3 Terrestrial: New Simulations

with Jupiter (stationary) | with Jupiter migration | w/ Jupiter migration (long)









Colors: Water fraction

Longterm N-body simulations
about 2000 bodies at start
about 10 M_{Earth} in $[0.5, 5.0]$AU
(Raymond et al., 2006-2007)

Problems:

- high eccentricity
- large Mars

Improvements

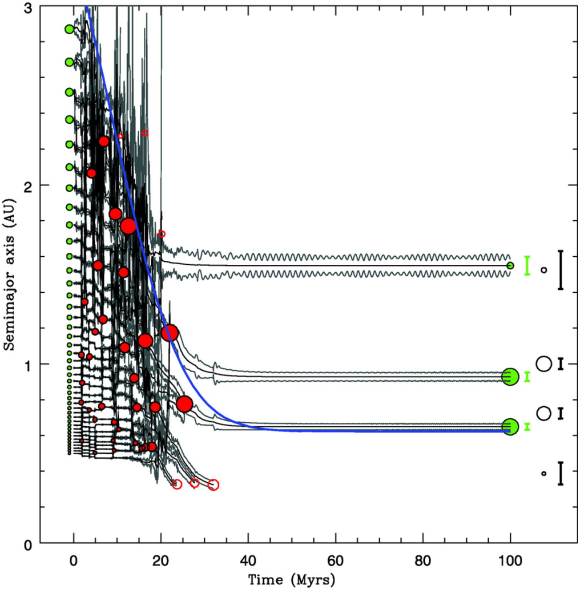
Too high eccentricities: Need dissipative process

- Planetesimals: left over from formation
  - reservoir filled by collision
  - damp e and i
  - are accreted by planet in *clean-up* phase,
- gas disk: left over from formation
  - damps e and i via tidal forces

Problem with all processes:
- Collisions of oligarchs require excentric orbits,
- but damping processes reduce e: contradiction!

Possible solution: Dynamical Shake-Up Model
(Nagasawa, Lin, Thommes; 2005, 2008)
Idea: Planets are still embedded in gas disk - e small and too few collisions
But e can be excited by secular resonances, i.e. precession of the apsidal line, $g_{planet} = d\varpi_{planet}/dt$, of the growing planet equals that of Jupiter $g_{\rm Jup}$ (or Saturn).
then resonance condition: $\Rightarrow$ increase of eccentricity (and more collisions)
disk influence diminishes with time.

2.3 Terrestrial: Shake-up example



Green Planets
Mass $\propto$ size

Blue Line: position of resonance (ν_5):
$g_{\rm planet} = g_{\rm Jup}$
Resonance 'pushes' planets inward

● coalescence
○ scattered out

right: final masses
with radial variation

○ Solar System

(Thommes et al., 2008)

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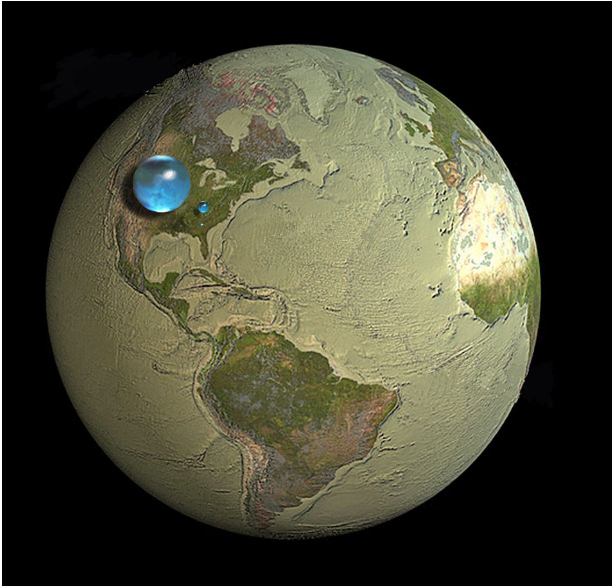
(World Water Day 2014, Lesley Simpson)

Fraction on Earth



Large sphere:
($D = 1400$km)
Total water content
of the Earth
$M_{H_2O} = 0.02\% M_{\rm Earth}$

Middle sphere:
($D = 272$km)
Total freshwater

small sphere:
($D = 57$km)
in lakes & rivers

in mantle:
10 times as much ?
Expt. to check solubility
of
H_2O in rocks

2.4 Water: Origin

At 1AU, the disk temperature is above the condensation temperature
$\Rightarrow$ water is present only in gaseous from

2 basic ways to gain water:
Water condensed on embedded dust grains and is directly incorporated into the Earth
Or material directly accreted from outside (throughn in by Jupiter)
see previous simulations (by Raymond ea.)

But initial Earth very hot (by collisions) $\Rightarrow$ evaporation ?

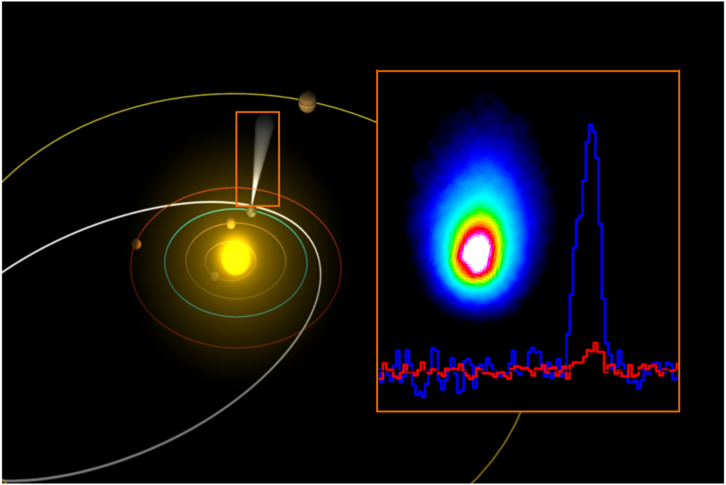
If later deposition of water ?
- Comets: Dirty snowballs
- Asteroids (see meteorites)

Here, clarification by isotopic composition of water:
- measure: D/H-ratio (Deuterium/'Protium')
  Deuterium already made in big bang, later only destroyed.

  Partially inconclusive results

Recent measurent: D/H identical on Earth and Vesta meteorite (Sarafian ea. 2014)

 D/H measurent at comet Hartley 2

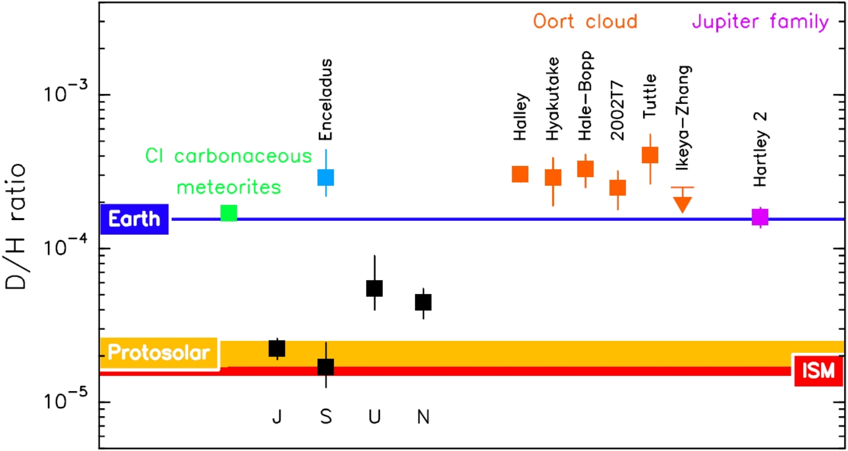


(Herschel Team, ESA, 2011)

shortest distance to Earth $\sim$ 0.13 AU = 20 Mio. km, in October 2010

In Spectrum: blue: $H_2{}^{18}O$ and red: HDO

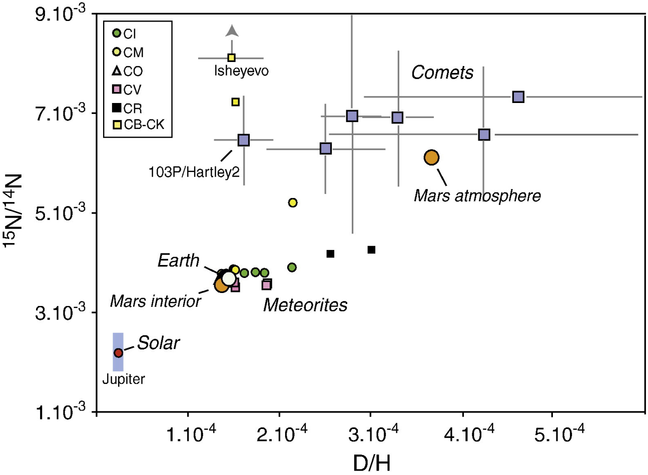
2.4 Water: D/H in Solar System objects



(Herschel Team, ESA, 2011)

⇒ Jupiter-type comets have D/H as the Earth, Oort-cloud-comets not
(Note: D/H is particle number ratio, not mass ratio) $(D/H)_{Earth} = 0.015\%$

 But: additional elements



(Marty, 2012)

Ratio: $^{15}N/^{14}N$ against D/H

$\Rightarrow$ Origin of water not complety clarified !

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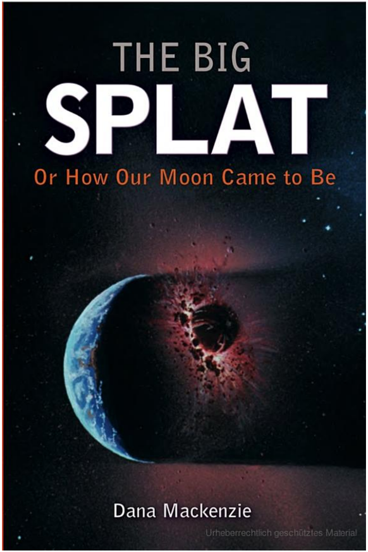
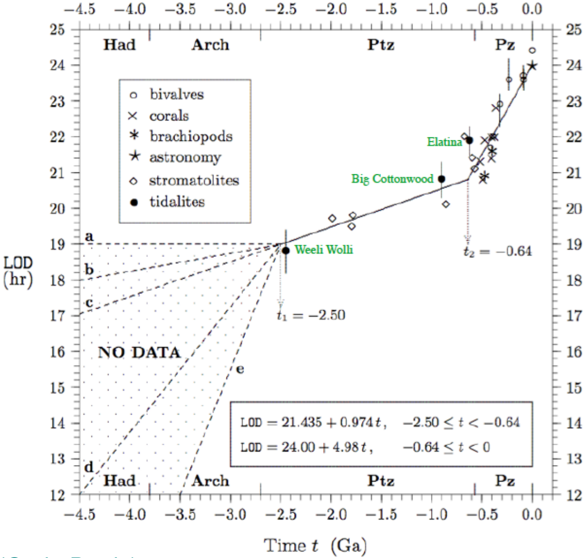


(A book title, 2003)

Change on Earth rotation (sediments)



(Carlo Denis)

Graphics:
Length of Day (LOD, hr)
vs. Time (10^9 yrs)

before 600 Mio. yrs:
(Dinosaurs)
length of day =
19-20 hrs

before 2 bil. yrs:
length of day:
19-20 hrs

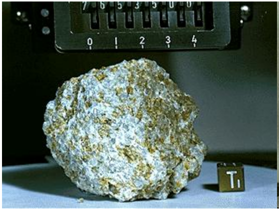
kink:
Continental drift ?

length at beginning:
16-17 hrs

2.5 The Moon: Age of lunar surface

Analyses of rocks from Apollo missions

Mare: few craters, relative young
3.1 - 3.8 bil. yrs old
late heavy bombardement,
example basalt



Crater: 4-4.5 bil. yrs,
1. example magnesium rich
rich in olivine, pyroxene

2. example anorthosite
(Feldspat Plagioklas)

radioactive age determination:
cristallisation of the magma ocean before 4.5 bil. yrs,
Age of Solar System: approx. 4.57 bil. yrs,
Moon about 30-40 mio. yrs younger than Earth

Basics of Formation

Features to explain:

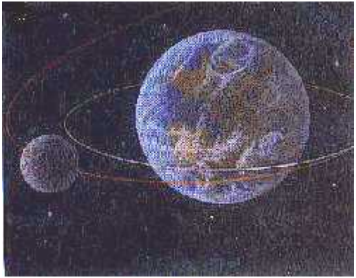
- Angular momentum of Earth-Monn system (very high compared to other satellite systems)
- very iron content (low lunar density)
- late formation (30 mio. yrs after Solar System origin)
- Oxygen isotopic composition identical to Earth
- volatile elements have lower abundance

4 suggested formation scenarios:

- Capture
- Double planet
- Fission
- Impact

Szenarios I

a) Capture



- not very likely
- dynamically difficult
- element abundances

b) Double planet

- inclination of Earth/Moon
- difference in density
- angular momentum

Szenarios II

c) Fission

+ low iron abundances
+ Oxygen fraction
- neede 2.5 hrs Earth rotation
- volatile elements

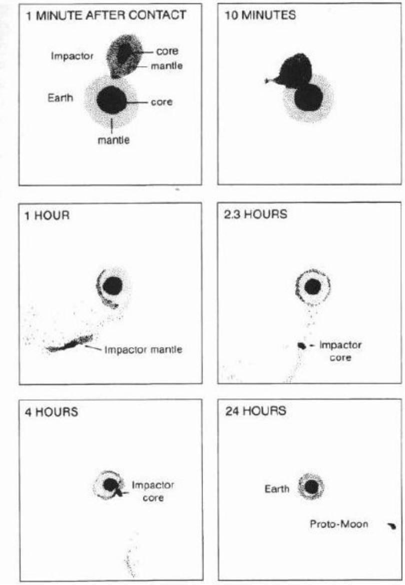
d) Impact

+ density difference
+ oxygen
+ volatile elements
+ angular momentum

Impact theory: (Hartmann & Davis, 1975; Cameron & Ward, 1976)

- predominant theory today
- probability of impact
- need impact of mars sized body: Theia

The impact scenario

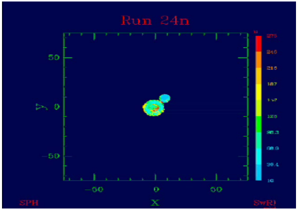


(Benz & Cameron, 1986-1991)

Simulation:
Smoothed Particle Hydrodynamics

color coding:
heating of the material

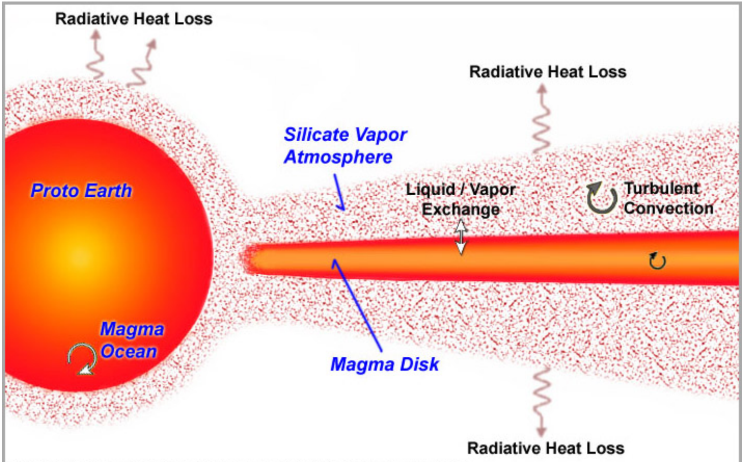
Simulation time: 24 hrs,
Earth rotation at end: 5 hrs



(Canup & Asphaug, 2001)

Problem:
- identical oxygen isotope abundances Moon-Earth
- But: Moon is composed only of Impactor material

Lunar accretion disk

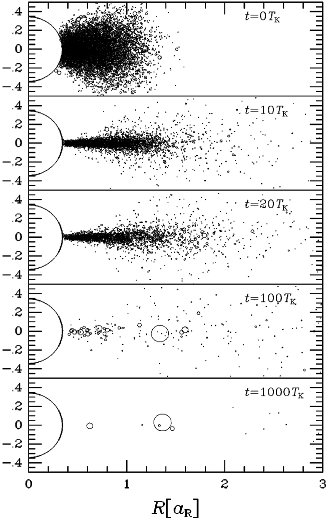


(PSRD graphic based on Pahlevan and Stevenson, 2007, *EPSL*, v. 262, p.438-449, Fig. 3.)

Well mixed materials in proto-lunarer accretion disk Only works well for volatile elements not for refractory but Tungsten and Titan abundances also identical (Touboul ea. 2007, Chang ea 2012) $\Rightarrow$ Still problems

Moon formation

N-body accretion simulations for moon formation



(Kokubo ea. 2000)

Unit of time:
Period at Roche limit ($a_R = 2.9R_{\oplus}$)
here about 7 hrs.

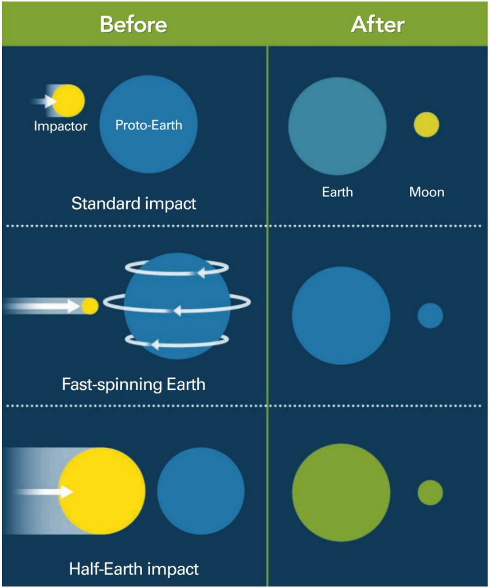
unit of length:
Roche limit a_R

circles:
proportional to particles size

Formation time of Moon:
about 1 month

Problems:
- spreading of material
- Mass in disk
- small initial distance
- Moon initially hot
  (shrinking upon cooling, cracks)

Variations of model



(Clery, Science, 2013)

Schematic classification of impact

Top:
Standard → Moon has different composition as Earth (but using velocity impact (Reuffer, 2012) → Moon has only small part of Theia

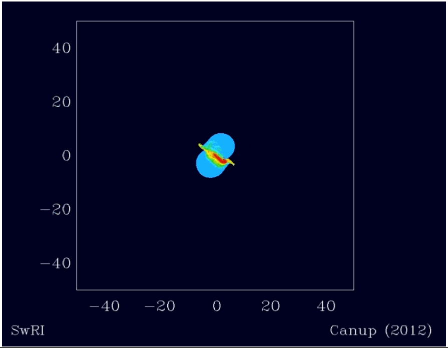
Middle:
Fast rotating Earth ⇒ well mixed (Cuk & Stewart, 2012)

Bottom:
Large body ⇒ very good mix of both material (Canup, 2012)

Impact of large body

Smoothed Particle Hydrodynamics Simulation (300.000 particles)
Two bodies with about 45 and 55% mass of todays Earth
with more angular momentum (a.m.) than todays sstem
(a.m. loss by 'evection resonance')
(allows for more freedom in impact parameter)

Color coding: temperature of material (from 2000 to over 6440 K)
about 3 lunar masses remain in disk around Earth $\rightarrow$ moon formation
(Canup, 2012)

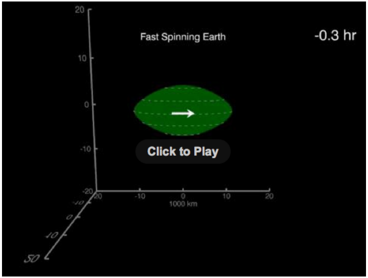


2.5 The Moon: Fast rotating Earth

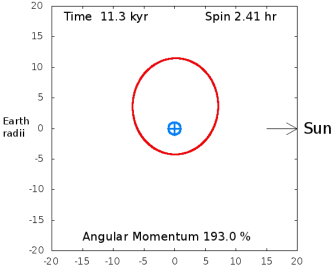
Smoothed Particle Hydrodynamics Simulations
Two bodies with Earth and Mars mass
with more a.m. than todays systems (Earth rotates faster)
- allows more freedom with impact parameter
  (a.m. loss by 'evection resonance')

about 2-3 lunar masses remain in disk around Earth $\rightarrow$ moon formation
(Cuk & Stewart, 2012)

The impact



Evection Resonance $(\omega_{prec} = \Omega_{♁})$



Still an active area of research !