From Dust to Planetesimals

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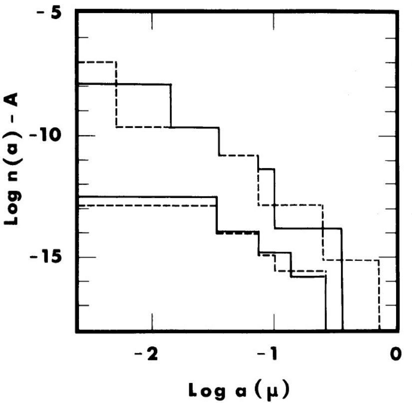
- 1.1 Context
- 1.2 Initial Growth
- 1.3 Dust dynamics
- 1.4 Dust concentration
- 1.5 Statistics

(Smithsonian)

Measure interstellar Extinction
Size distribution (MRN)

$n(a) \propto a^{-3.5}$



(Mathis, Rumpl, Nordsieck, 1977)

Typical sized around $0.1 - 1\mu$m

Ingredients II - Protoplanetary Disk

MMSN: Minimum Mass Solar Nebula

- Augment planet mass with gas to solar abundance
- split into rings
- spread material therein

using $r_{AU} = r/(1AU)$

$$\Sigma_g(r) \approx 1700 \, r_{AU}^{-3/2} \, \mathrm{g/cm^2} \qquad (1)$$

(Hayashi, 1981)

Temperature (only stellar irradiation)

$$T(r) \approx 280 \, r_{AU}^{-1/2} \, \mathrm{K} \qquad (2)$$

Total Mass:

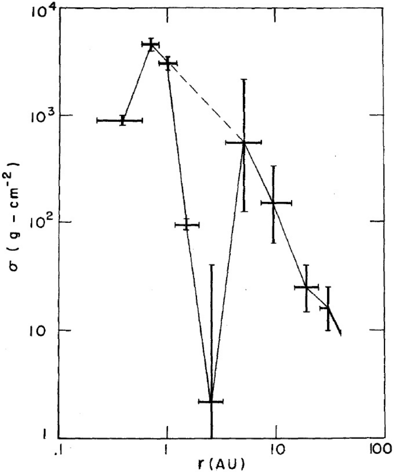
$\approx 0.01 - 0.05 M_\odot$

total angular momentum:

$\approx 3 \cdot 10^{51} - 2 \cdot 10^{52}$ g cm^2 s^{-1}

typical model for protoplanetary disks

mass distribution in Solar System

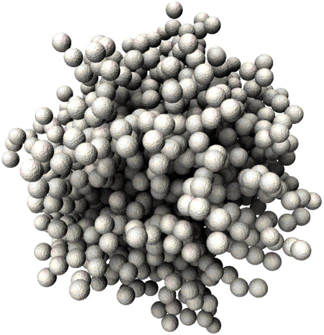


(Weidenschilling, 1977)

1.1 Context: Growth Modes

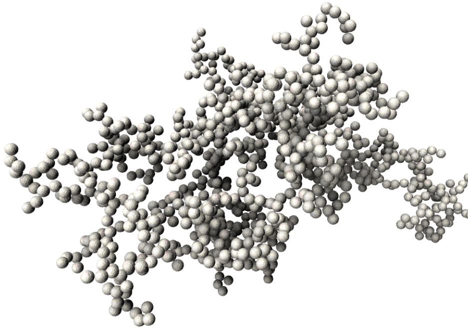
<u>BPCA</u>

Ballistic Particle-Cluster Agglomeration:
Connect individual particles



<u>BCCA</u>

Ballistic Cluster-Cluster Agglomeration:
Connect whole clusters

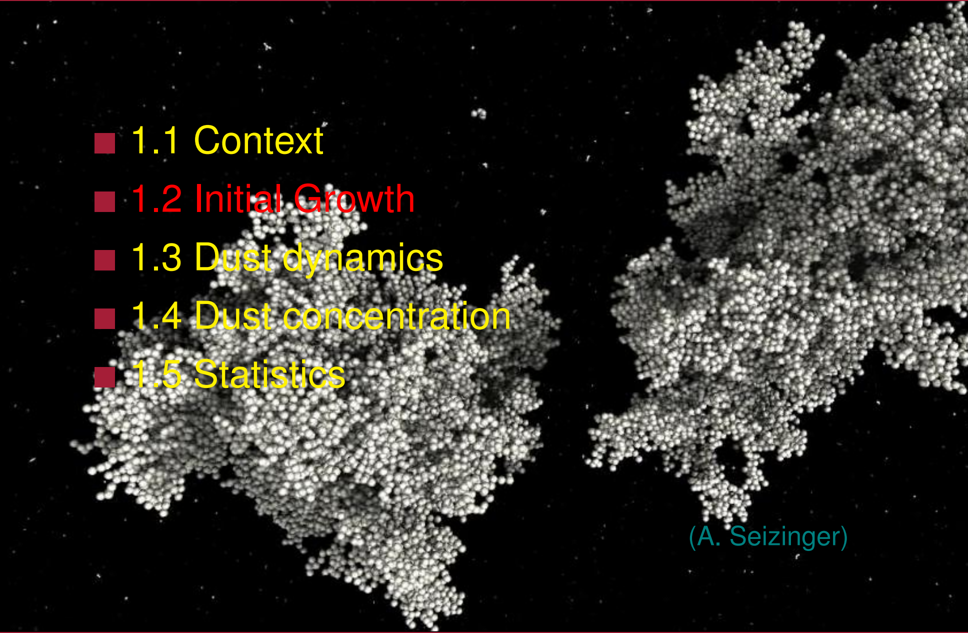


(A. Seizinger)

Example with $N = 1024$ Particles (here 100% Sticking)

Sticking by: Van der Vaals, enhancement for ice, organic compounds

Organisation



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(A. Seizinger)

Processes

Growth of particles through adhesive mutual collisions

Depends on the following properties

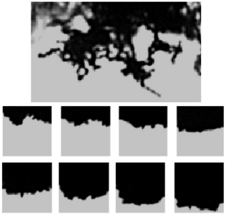
- Sticking probability
- velocity dependence
- friction coefficicients
- compactification
- collisional outcome

Approaches:

- Experiments
- Simulations

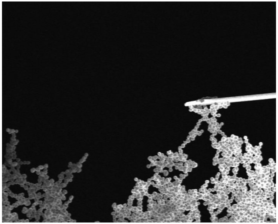
Experimental Dust Growth

Laboratory-Experiments: μm-sized particles, (J. Blum, & G. Wurm)



Blum & Wurm 2000

Measure pull off force

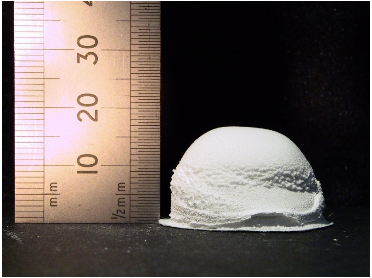


in Vacuum, zero gravity
- in lab
- fall Tower (Bremen)
- parabolic flights
- space station

initial Fractal Growth

works up to cm-sizes

Dust Cake (for expts.)

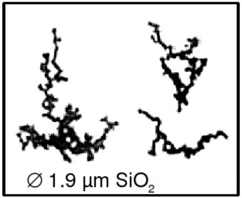


Filling factor 15%

Low velocity regime

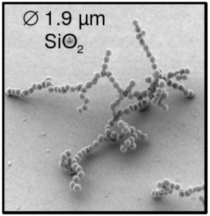
Test of different experimental conditions (with $v_{rel} \approx 10^{-4} - 10^{-2}$ m/s)

Gas turbulence



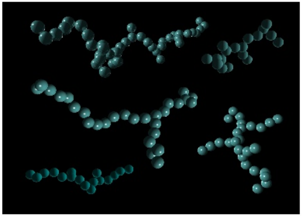
Wurm & Blum 1998

Differential sedimentation



Blum et al. 1998

Brownian motion



Blum et al. 2000

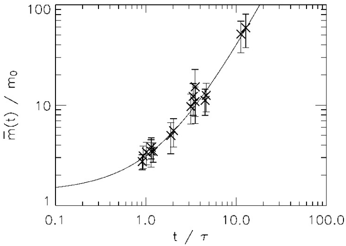
Fractal Growth

Mass (m)-Size (s) Relation:

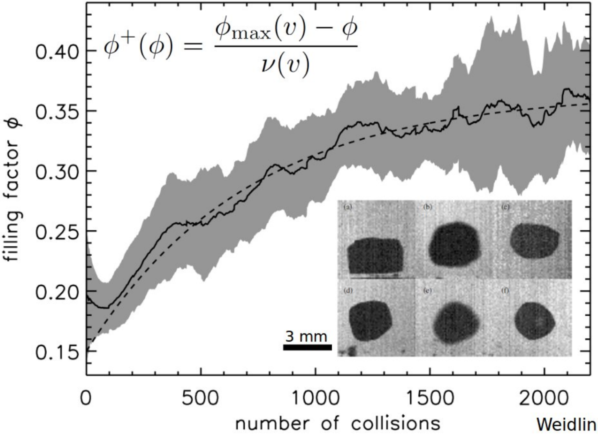
$m \propto s^{D_f}$ mit $D_f \leq 2$

Typical here: $1.4 \leq D_f \leq 1.8$

Approximative: $m(t) \propto t^{1.7}$



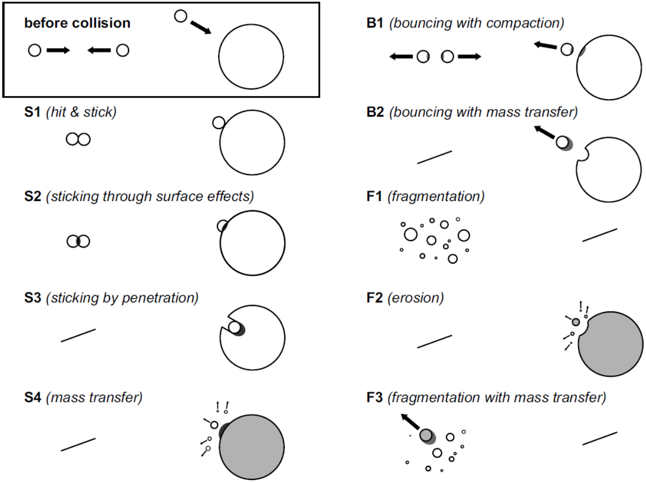
Consecutive collisions lead to compactification of aggregates
The filling factor is given by $\phi = \rho_{mat}/\rho_{aggregate}$



Weidling et al., 2009

Outcomes of experiments

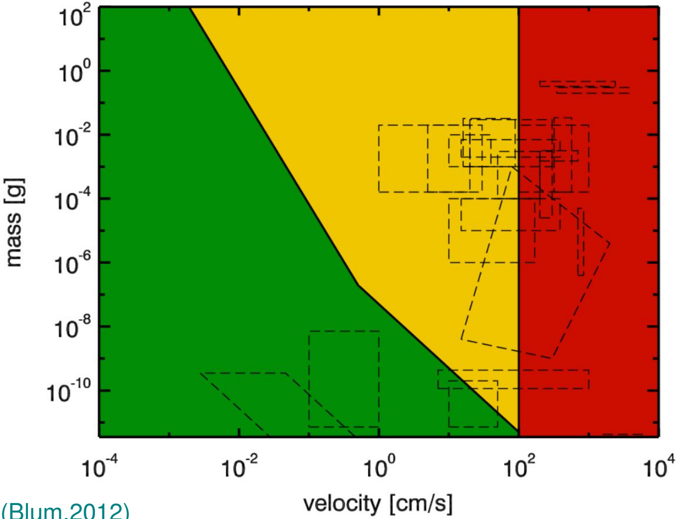
Classification scheme



(Güttler&Blum,2010)

Possible growth regimes

Investigate collisions of equal mass particles
Green: Sticking, Yellow: Bouncing, Red: Fragmentation



(Blum,2012)

Sticking:
for small and slow particles

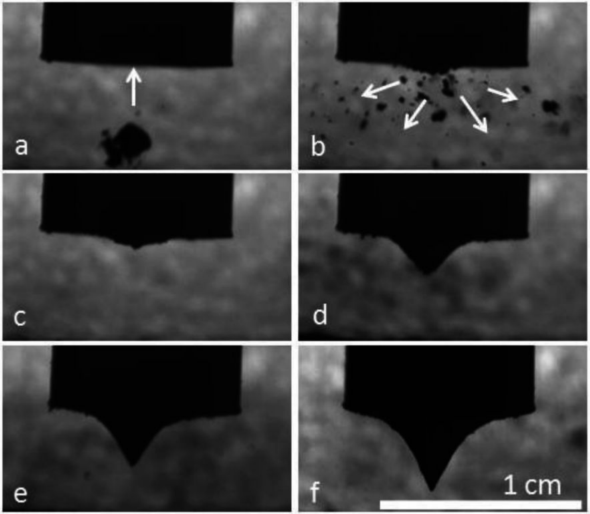
Bouncing:
leads to compression of particles up to $\phi \approx 0.4$

Fragmentation:
above critical velocity $u_{frag} \approx 1$m/s

Dashed boxes:
regions covered by experiments

Growth by mass transfer

Possible growth of particles by transferring in high speed collisions.
Transfer mass from a small projectile to a large target



(Credit: Stefan Kothe, from Johansen ea. 2014)

mm-sized projectiles onto cm-sized target (at top)

Mono disperse spheres: $d = 1.5\mu$m
Here speed: 4.2m/s
Growth for much higher impact speeds, up to a few 10 m/s

see also (Wurm ea. 2005)

Numerical modeling

Model the motion of N Particles with Newtonian Dynamics:

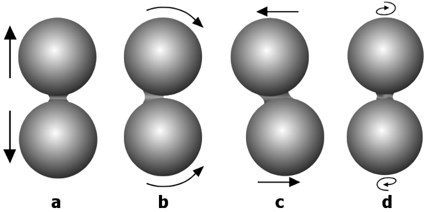
Molecular Dynamics type of modeling with special interactions, that may be divided into the following types:

a) Compression/Adhesion (Johnson *et al.*, 1971)

b) Rolling (Dominik & Tielens, 1995)

c) Sliding (Dominik & Tielens, 1996)

d) Twisting (Dominik & Tielens, 1996)



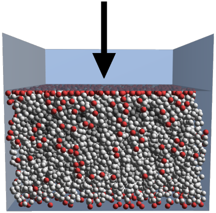
Forces and torques of those types of interaction can be derived from *corresponding potentials* (Wada *et al.*, 2007)

Advantage: Can track in detail the energy evolution. Calculate continuum material parameter

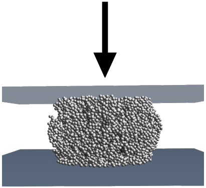
Calibration with experiments

Calibrate numerical parameter through quasi-static numerical compressing

a) Multi-directional compression

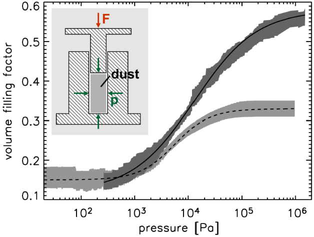


b) Uni-directional compression



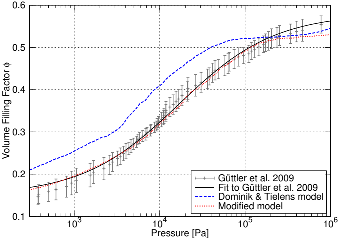
Experimental Setup & Results

(Güttler ea. 2009)



Numerical Results (case a)

(Seizinger ea. 2012)

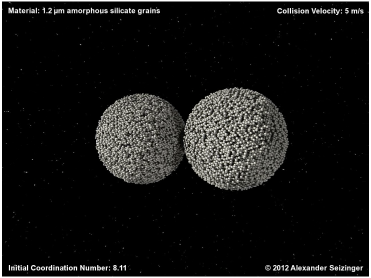


Bouncing/Fragmentation

Collide two CPE (Close Packing with Extraction) aggregates
with coordination number $n_c = 8.11$ and filling factor $\phi = 0.52$
Two collision velocities

$v_{rel} = 2$ m/s

$v_{rel} = 5$ m/s

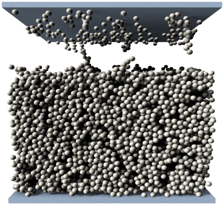


Outcome depends on n_c and ϕ
Only overall filling factor experimentally measurable, not n_c

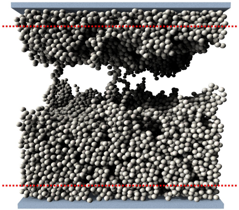
Continuum parameter

Setups for measuring tensile and shear strength as a function of the filling factor (Seizinger ea. 2013) (Difficult to measure experimentally)

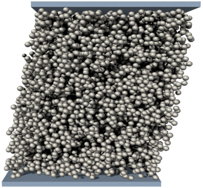
No wall glue



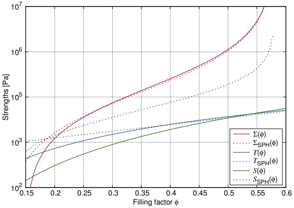
With wall glue



Measure shear strength

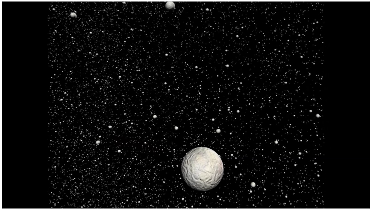


Comparison with SPH values

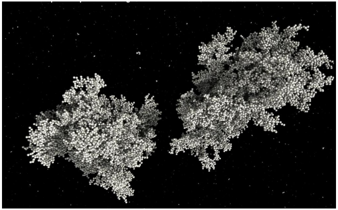


Numerical: Cluster-Cluster Collisions

Initial Growth Phase



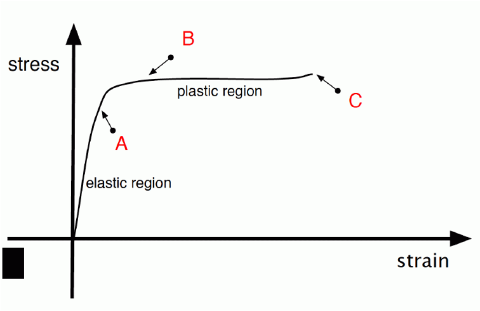
Collision with different speeds



Animations by Alexander Seizinger (Tübingen)

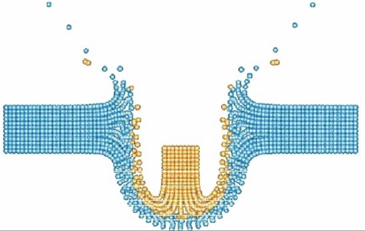
Continuum Approach

For larger particles:
Better Method: SPH
(Smoothed-Particle-Hydrodynamics)
Hydrodynamical-Equations
augmented with:
Elasto-plastic Model (A-B)
Including:
Fractures (C)

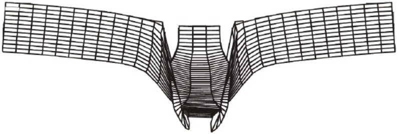


Example: Impact into Al-Plate
(Rabczuk, 2002)

SPH Simulation:

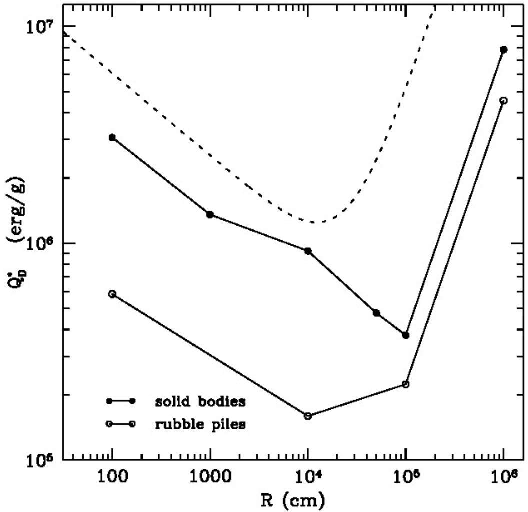


Finite Element Simulation:



Destruction Threshold

Catastrophic Disruption Threshold: (that specific energy Q, at which the largest intact particle has = 1/2 Targetmass)



Collisions of Basalt spheres with $v_{rel} = 20$m/s (dashed: 3km/s)

$$Q_D = \frac{1}{2} m_{proj} v_{proj}^2 / M_{target}$$

Weakest Bodies:
$R \approx 100$m

●: Stones
○: Rubble Pile

(Benz, 2000)

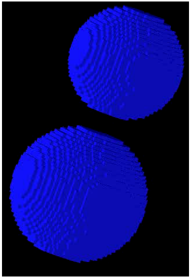
(In protoplanetary Disk: $v_{rel} \approx 10 - 100$m/s, depending on mass, size, location ..)

Check growth of planetesimals by collisions/accretion

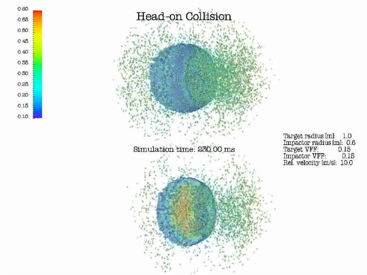
SPH (Smoothed-Particle-Hydrodynamics) using 250,000-500,000 particles

Here: Elastic-plastic strength model, formation and evolution of cracks

2 Basalt Spheres:



Porous Objects: $r = 6, 10\text{m}$, $v_{rel} = 10\text{m/s}$



(cp. small objects in Solar System)

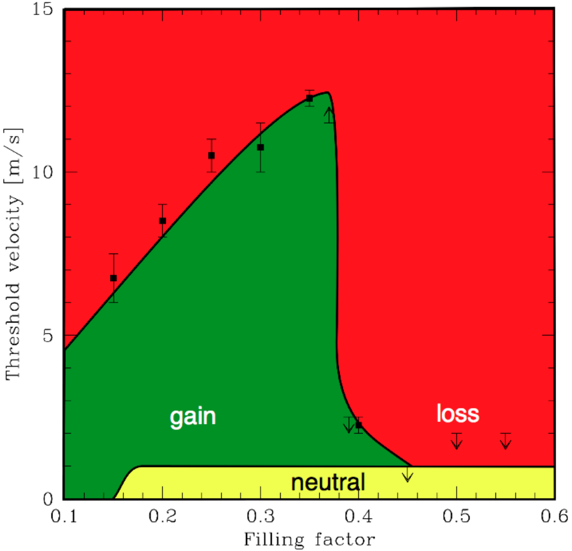
(Schäfer, Geretshauser, Speith, Meru; Tübingen)

(Geretshauser ea., 2010, 2011a,b)

$\Rightarrow$ Need porous bodies !

Growth Regime

Collide two objects with $R_{target} = 10$ cm and $R_{projectil} = 6$ cm



Symbols:
Numerical Results
(Fragmentation velocity)

Red: Fragmentation
Green: Sticking (Growth)
Yellow: Bouncing

(Meru ea. 2013)

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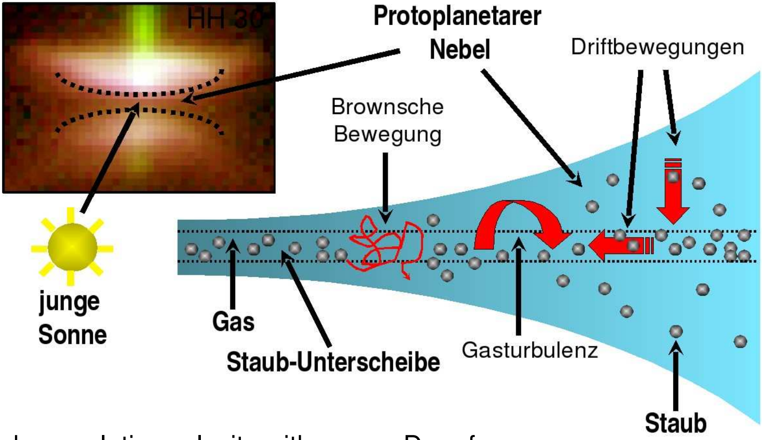
(Haboob Over Phoenix, Arizona)

Particle Dynamics in Disk

Particles need relative Velocity for mutual collisions. Origin
- Brownian Motion
- Vertical Sedimentation (Settling), radial Drift
- Turbulence in the Disk



(J.Blum)

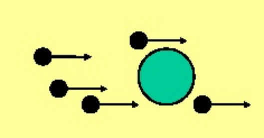
Particles have relative velocity with gas ⇒ Drag forces

Dust-gas interaction

Particle with radius a in gas with density ρ_g, mean free path λ_f, sound speed (thermal velocity) c_s, and relative velocity $v_{\rm rel}$ relative to gas

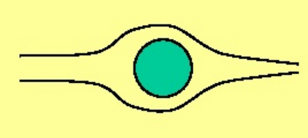
Epstein regime $\lambda_f \gg a$

$$\vec{F}_D = -\frac{4\pi}{3}\rho_g a^2 c_s \vec{v}_{\rm rel} \qquad (3)$$



Hydrodynamic (Stokes) regime $\lambda_f \ll a$

$$\vec{F}_D = -\frac{C_D}{2}\pi a^2 \rho_g v_{\rm rel} \vec{v}_{\rm rel} \qquad (4)$$



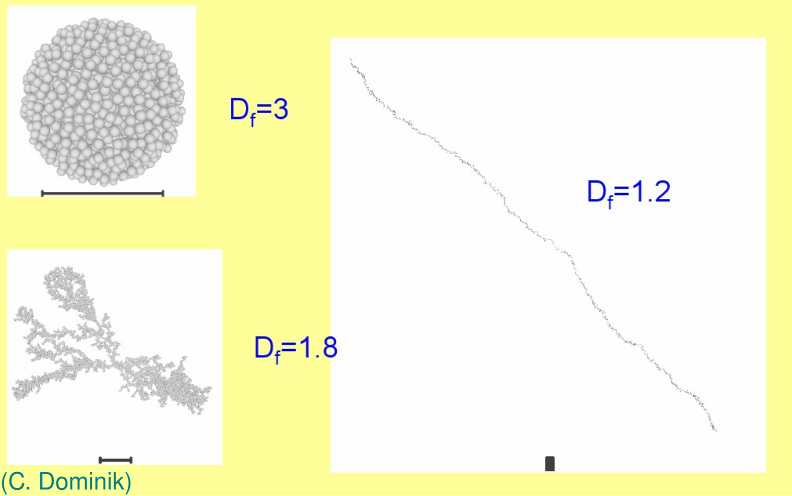
Coefficient of resistance, C_D, (sph. particles), see (Weidenschilling, 1977):
Typical mean free path at 1AU: $\lambda_f \approx 1$cm !

Equation of motion for dust particles (with mass m_d and density ρ_d)
(only friction part)

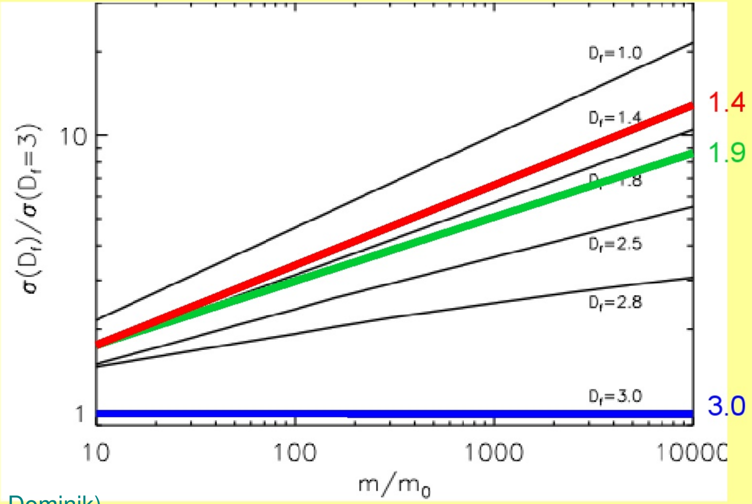
$$m_d \, \ddot{\vec{r}} = -\vec{F}_D \qquad (5)$$

(*Stopping-Time*) = Time, where $v = v_{rel}$ drops by $1/e$: $t_{fric} = |v/\dot{v}| = m_d v / F_D$

Mass/Surface ratio I



In the Epstein regime t_{fric} depends on the Mass/Surface ratio of dust particle.
The fractal dimension D_f determines the mass increase with radius, a:
$M \propto a^{D_f}$



(C. Dominik)

Plotted is the surface, σ, of an object with D_f to that of an object with $D_f = 3$
The fractal dimension D_f determines the mass/surface ratio

Values for $t_{\rm fric}$ ($=t_e$)

- Epstein regime

$$t_e = 200s \left(\frac{\rho_d}{2 \frac{\rm g}{\rm cm^2}} \right) \left(\frac{10^{-11} \frac{\rm g}{\rm cm^3}}{\rho} \right) \left(\frac{a}{1 \mu{\rm m}} \right) \left(\frac{1{\rm km/s}}{\rm c_s} \right)$$

- Hydrodynamic regime

$$t_e = 2.7{\rm yr} \left(\frac{\rm a}{1{\rm cm}} \right) \left(\frac{\rho_d}{2 \frac{\rm g}{\rm cm^3}} \right) \left(\frac{24}{\rm C_D} \right) \left(\frac{10^{-11} \frac{\rm g}{\rm cm^3}}{\rho} \right) \left(\frac{1{\rm m/s}}{\rm v} \right)$$

(C. Dominik)

In the Epstein regime (small particles) they are well coupled to the gas.

Drift of dust particles

In a disk, the gas experiences pressure forces, i.e. moves slightly sub-Keplerian. But particles move on Keplerian orbits. This gives rise to a drag force:

$$\vec{F}_D = -\frac{\vec{v}_\mathrm{d} - \vec{v}_\mathrm{g}}{t_\mathrm{fric}} \tag{6}$$

In equilibrium ($d/dt = 0$) this has to balance the pressure gradient of the gas, and we obtain

$$\vec{v}_\mathrm{d} - \vec{v}_\mathrm{g} = \frac{t_\mathrm{fric}}{\rho_g} \nabla p \tag{7}$$

$\vec{v}_\mathrm{rel} = \vec{v}_\mathrm{d} - \vec{v}_\mathrm{g}$ always points in the direction of the positive pressure gradient.

$\Rightarrow$ particles collect in pressure maximum   (if that exists)

In particular: The velocity difference in azimuthal direction leads to a drag force, and a loss of angular momentum. i.e. to an inward drift of particles.   And additional vertical settling.

Radial Drift I

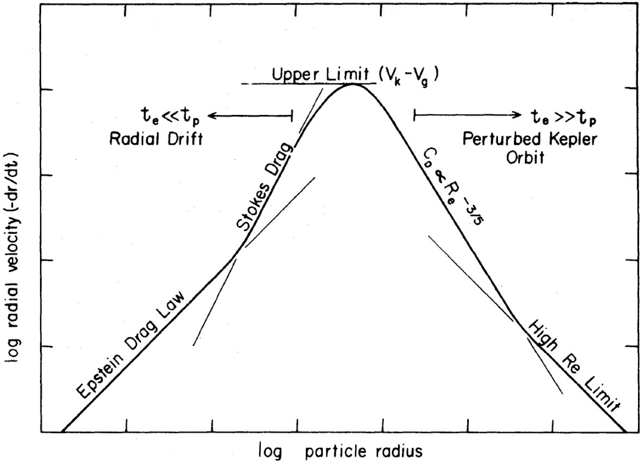


Diagramme: log(velocity) vs. log(particle size)

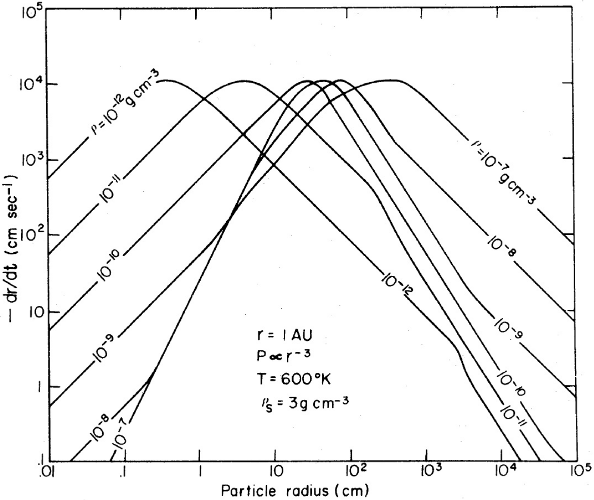
$t_e = t_{\rm fric}$ (Stopping time)

$t_p = t_{\rm orbit}$ (Period)

(S. Weidenschilling, 1977)

different drag regimes: forces change slope
small particles: well coupled to the gas $\Rightarrow$ small drift rates
large particles: decoupled from gas $\Rightarrow$ small drift rates

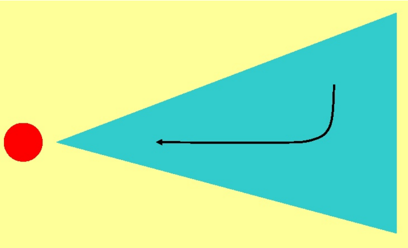
Radial Drift II



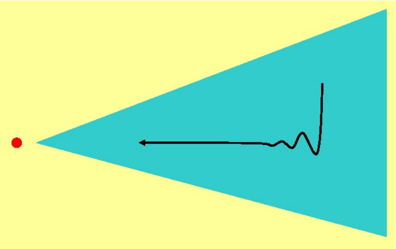
Curves for different gas densities (S. Weidenschilling, 1977)
maximal drift rates for dm to meter-sized particles: 10^4cm/s
i.e.: Drift from 1AU to Sun in only 100 years !

Vertical Settling and Oscillation

Trajectory for small particles



Trajectory for large particles



(C. Dominik)

For $t_{\rm fric} \ll 1/\Omega_K$ is escape velocity $v_{\rm settle} = -z\Omega_K^2 t_{\rm fric}$ is quickly reached.
Define *Settling*-time (Epstein regime) (with $c_s = H\Omega_K$, $\Sigma = \pi/2\,\rho_0 H$):

$$t_{\rm settle} \equiv \frac{z}{|v_{\rm settle}|} = \frac{2}{\pi}\frac{\Sigma}{\Omega_K \rho_d\, a_d}\, e^{-z^2/2H^2} \qquad (8)$$

Note: strong density dependence of $t_{\rm settle}$, very short in upper layers

- μm-sized particles need million years for *settling*
- Golfball-sized only about $10^3 - 10^4$ years.

Note: approximation for a laminar disk is not very good.

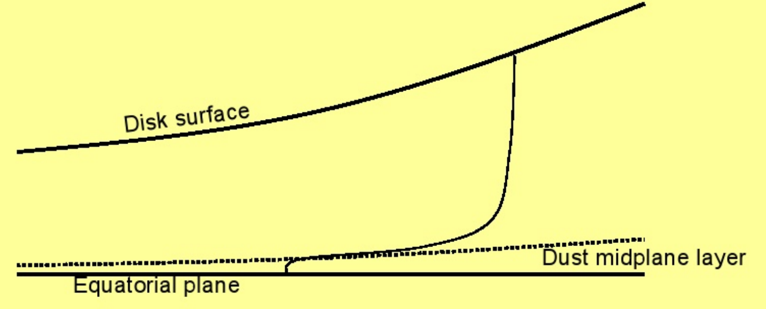
 Organisation

- 1.1 Context
- 1.2 Initial Growth
- 1.3 Dust dynamics
- 1.4 Dust concentration
- 1.5 Statistics

(Haboob Over Phoenix, Arizona)

Thin dust layers

Dust particles settle to midplane and form thin dust layer



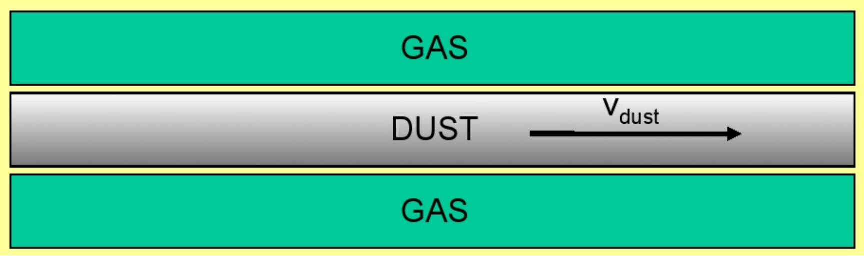
(C. Dominik)

For a dust thickness $H_d \approx 0.01 H$ is $\rho_d \approx \rho_g$ (dust/gas-ratio = $1/100$)
then dust determines the motion of the gas (not the other way around)

$\Rightarrow$ Possibility of *Instabilities*:

- Kelvin-Helmholtz (through relative tangential motions of dust and gas) (origin of turbulence in disks ?)
- gravitational instability (of the thin and dense dust layer only) (circumvene the dust growth process ?)

Gas and dust particles have relative velocities

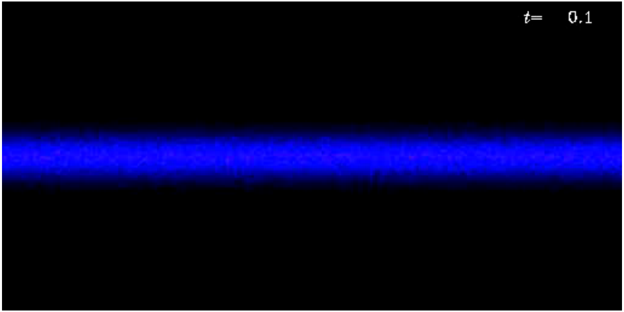


(C. Dominik)

Discontinuous v_φ-velocity (in z-direction)
particles have Keplerian velocity v_K, gas slightly lower (because of pressure gradient)
$\Rightarrow$ Kelvin-Helmholtz-Instability

KH-Numerical Simulation

Particles (cm-sized) in *laminar* purely hydrodynamic disk
2D-simulation (in $z - \varphi$ direction)



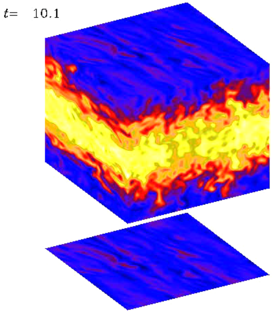
(A. Johansen)

Particles sink to midplane and drag gas with them
$\Rightarrow$ vertical gradient of v_φ-velocity of the gas
$\Rightarrow$ Kelvin-Helmholtz Instability

But: disk does not become globally turbulent
(possibility of concentration of particles in turbulent eddies ?)

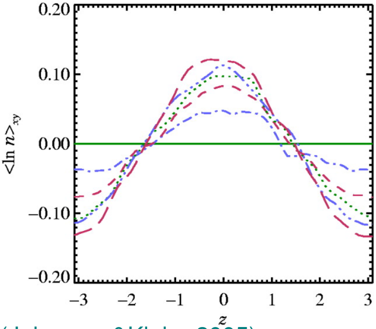
Dust in turbulent disk

dust particles
in *local shearing box*
Magnetohydrodynamical
simulation



(A. Johansen)

dust Sedimentation
plot log(dust density n) as function of z
from $t = 0 (n = 1)$ up to $t = 10$, $\Delta t = 2$



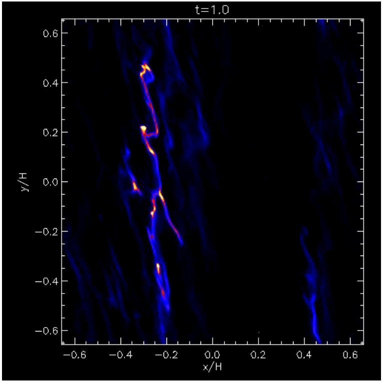
(Johansen&Klahr, 2005)

Particles sink to midplane and form dust layer
From vertical extension of layer calculate the Diffusion coefficient ($D_{\rm turb}$):
In equilibrium: Settling $\approx$ turb. Diffusion
Turbulent particle diffusion ($D_{\rm turb} = \alpha_{\rm diff} c_s H$) $\approx$ gas momentum diffusion (ν).

Planetesimal-Growth in turbulence

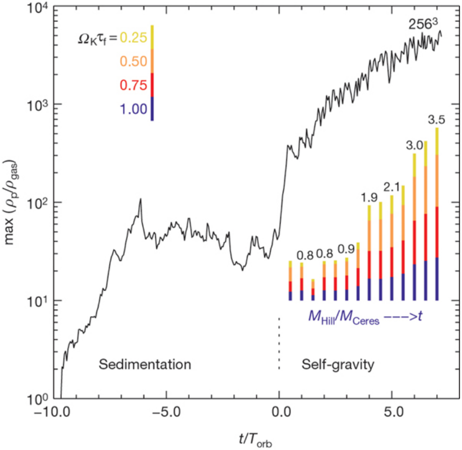
Planetesimal formation through Gravoturbulence

Magneto-Hydrodynamic-simulation with particles under gravitativer interaction



(A. Johansen)

Growth through gravitational instability of the dust particles $(m_{\rm tot} = 50 m_{Ceres})$
Growth: up to $3.5 m_{Ceres}$ in ≈ 7 Orbits

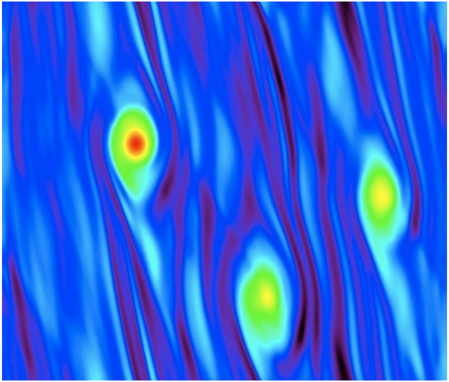


(Johansen et al., 2007)

Dust in vortices

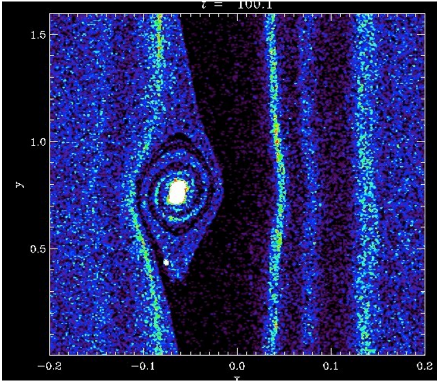
Vortices can form in accretion disks if the entropy gradient exceeds a critical limit

Vortices in gas disk



(Klahr, Raettig, Lyra)

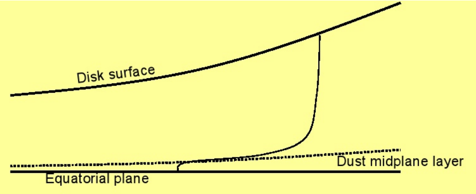
Dust concentration in vortices



Open questions: Lifetime of vortex, sheared apart, role of self-gravity

Gravitational instability - principle

The dust particles form a dense layer in midplane



(C. Dominik)

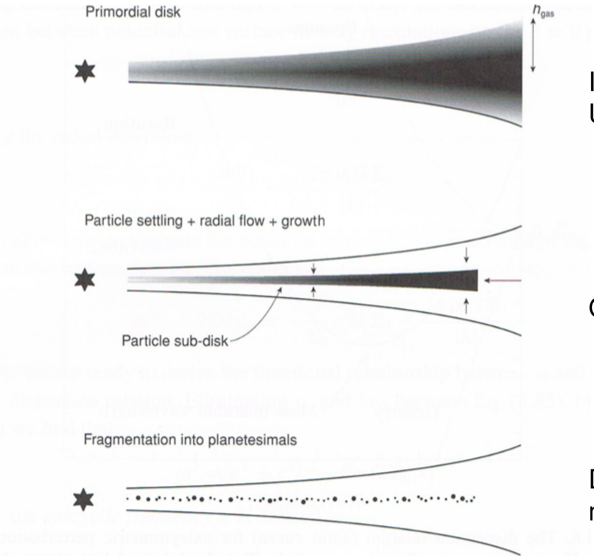
Treat particles as infinitesimal thin sub-disk, i.e. $\rho_d > \rho_g$. And investigate its stability properties using a linear stability analysis for a 2D-disk.

It follows (lecture 7) that an axially symmetric flow is stable for

$$Q \equiv \frac{c_s \kappa_0}{\pi G \Sigma_0} > 1 \qquad \text{(Toomre-criterion)} \tag{9}$$

pressure and rotation stabilize, gravitation destabilises
κ = epicyclic frequency, equals Ω_K
(important in galactic dynamics, non-axially symmetric: slightly higher Q)

Dust in PP-disk



Initially:
Uniform dust distribution

Gravitational Settling

Dense dust layer in midplane $\rightarrow$ planetesimals

(Armitage, 2010)

Application to dust disk

For typical pp-disk parameter

$$c_s = v_{disp} = 10\,\text{cm/s} \tag{10}$$

for the MMSN at about 1 AU:
Size of collapsing region: 10^8 cm
mass in collapsing region: 10^{18}g
$\equiv$ 6 km radius for $\rho = 3\text{g/cm}^3$
Timescale for collapse: < 1 year !

Attractive scenario: avoids radial drift problem

BUT:
adding a small turbulence $\rightarrow$ spreading of dust layer $\rightarrow$ possible stabilisation

Summary

Initially collisional growth
Several barriers (for cm to meter sized objects)

- Bouncing
- Fragmentation
- Drift

Possible solutions

- very porous objects
- mass transfer collisions
- icy material ($v_{crit} \approx 70$ m/s)
- condensation
- velocity distribution of particles

Organisation

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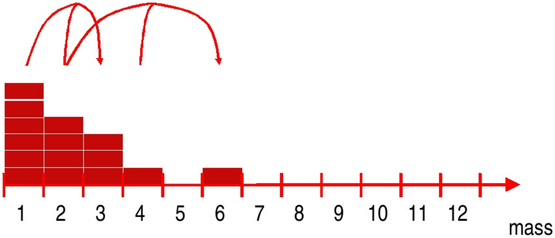
(Haboob Over Phoenix, Arizona)

Divide particles in mass bins. Mass changes in the individual bins through collisions with particles from other bins



(C. Dullemond)

Discrete distribution function N_i: Number of particles (/cm^3) with i monomers

Coagulation equation (in discrete form: Smoluchowski 1916)

$$\frac{dN_k}{dt} = -\sum_{i=1}^{n} s_{i,k} \Delta v_{i,k} N_i N_k + \sum_{i=1}^{k-1} s_{i,k-i} \Delta v_{i,k-i} N_i N_{k-i} \tag{11}$$

$s_{i,k}$ = Crosssection for collision between particles i and k

$\Delta v_{i,k}$ = mean relative velocity between particle i and k

n = Total number of bins

Problem: Need 10^{30} Bins!

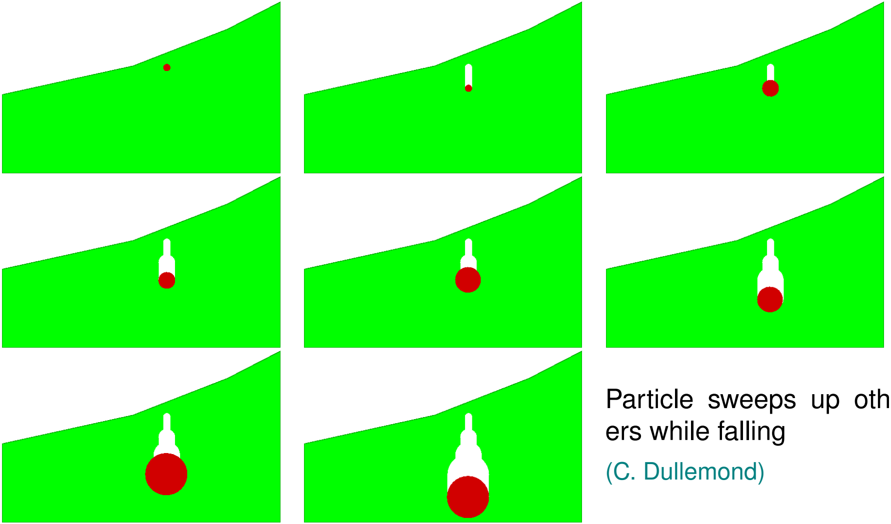
Number of particles is too large to treat all at the same time
Work with (continuous) distribution function $f(r, z, m, t)$
$f(r, z, m, t)\, m\, dm$ is dust mass density at position (r, z) and time t (in g/cm^3)

$$\left.\frac{\partial f(m)}{\partial t}\right|_{\mathrm{coag}} = -\int_0^\infty f(m')f(m)\sigma_c(m', m)\,\Delta v(m', m)dm' + \int_0^{m/2} f(m')f(m-m')\sigma_c(m', m-m')\,\Delta v(m', m-m')dm' \quad (12)$$

$$\text{with kernel} \quad K(m_1, m_2) = \sigma_c(m_1, m_2)\Delta v(m1, m_2) \quad (13)$$

Crucial ingredient: $\Delta v(m_1, m_2)$. One example ((Dullemond & Dominik 2005))
– Brownian motion: $\Delta v(m_1, m_2) = (8kT(m_1 + m_2)/(\pi m_1 m_2))^{1/2}$
– Differential vertical settling: $\Delta v(m_1, m_2) = |v_{\mathrm{sett}}(m_1) - v_{\mathrm{sett}}(m_2)|$
– Turbulence driven coagulation: ... (more complex)
and equation for vertical settling: $f(m)/\partial t|_{\mathrm{sett}} = ...$
Use numerically discrete massbins with $\Delta m \sim m$ (logarithmical)

Consider particle at (r, z) and follow vertical motion $z(t)$ and growth $m(t)$, while all other particles remain unchanged, schematically:

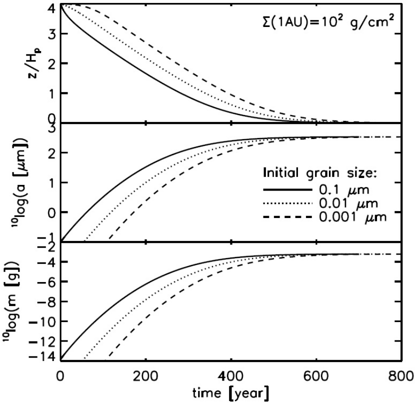


Particle sweeps up others while falling

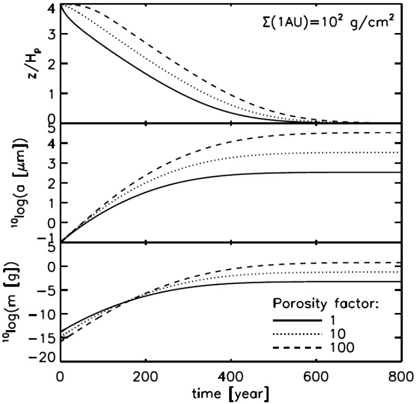
(C. Dullemond)

consider isothermal disk model with $M_* = 0.5 M_\odot$, $T_* = 4000$K, $R_* = 2.5 R_\odot$
at $r = 1$AU, with $\Sigma_{gas} = 10$g/cm^2 and $T = 204$K (Dullemond & Dominik, 2005)

dependence on initial size



dependence on initial porosity (cp. m/σ)

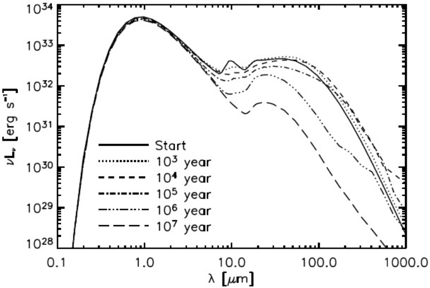


Fall- and growth-time very short $\ll$ life time of disk !
little dependence on initial size and porosity
Porous bodies grow somewhat larger (larger crosssection , σ_d)

small Rain from
Cumulus Congestus clouds

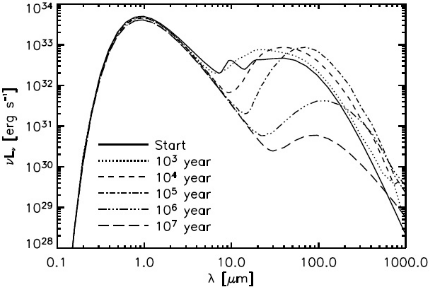
Hierarchical growth
for hale grains

(C.Dullemond)



Combine rings of 0.7 – 100AU
no radial mixing (initial phase)
calculate theoretical model spectra

Top: w/o turbulent mixing



Bottom: w/ turb. mixing

turbulent motion increases # collisions
very (too) short evolution time !
(Dullemond & Dominik, 2005)

Problem:
Theoretically the dust particles are not visible due to settling and growth but observed