41st Saas-Fee Course
From Planets to Life
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Lecture 5: Hydrogen Escape, Part 1

Prebiotic O_2 levels/
Kinetic theory of gases/
Jeans escape/
Nonthermal escape

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Why do we care about hydrogen escape?

- Most H comes initially from H_2O. Thus, when H escapes, O is left behind

  $\Rightarrow$ *terrestrial planets become more oxidized with time, even without biology*
- Atmospheric scientists got the prebiotic O_2 level wrong for many years before Jim (J.C.G.) Walker finally got it right
  - The reason they got it wrong was because they didn't understand hydrogen escape
- This problem is important, because it bears on the question of whether O_2 in an exoplanet atmosphere is a sign of life

Prebiotic O_2 levels—historical perspective

- Berkner and Marshall (1964, 1965, 1966, 1967) tried to estimate prebiotic O_2 concentrations
  - They recognized that the net source of O_2 was photolysis of H_2O followed by escape of H to space
  - These authors assumed that O_2 would build up until it shielded H_2O from photolysis

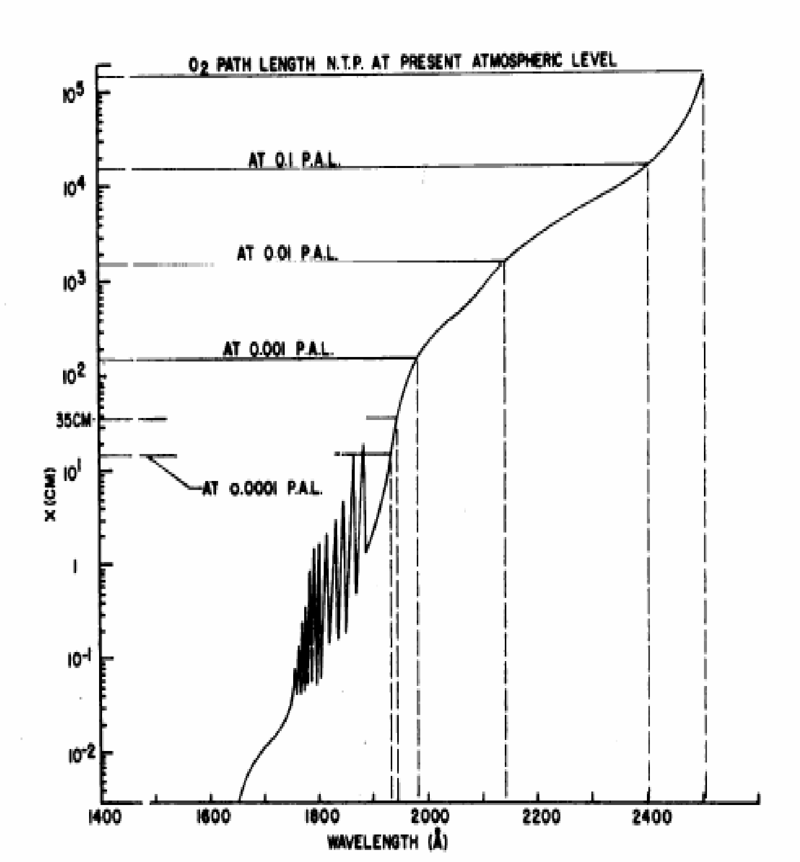
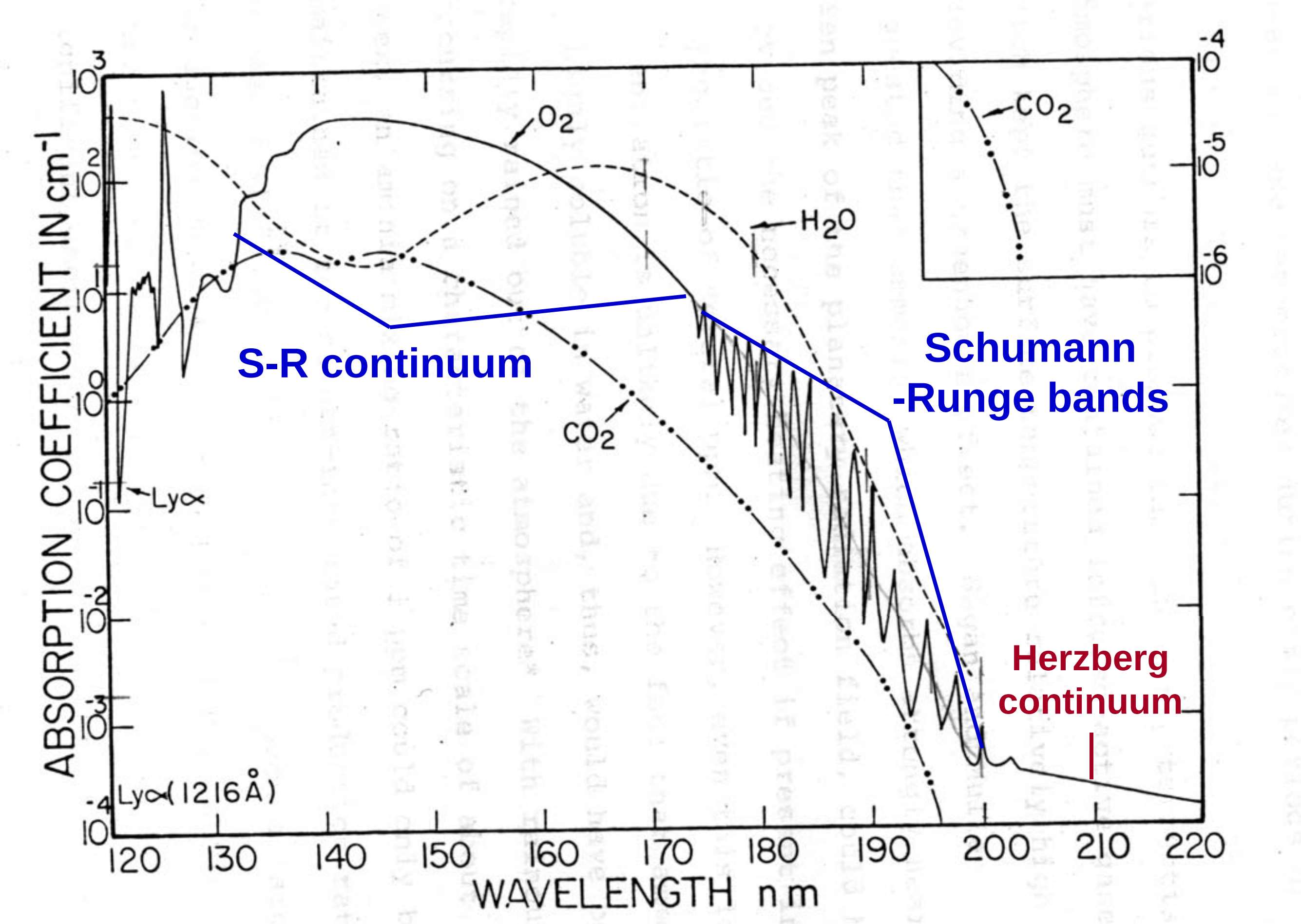


FIG. 5.—Thickness of O_2 required to absorb available UV to "extinction" [1 erg cm^{-2} sec^{-1} (50 Å)$^{-1}$].

UV absorption coefficients of various gases



Source: J.F. Kasting, Ph.D. thesis, Univ. of Michigan, 1979

Berkner and Marshall's model

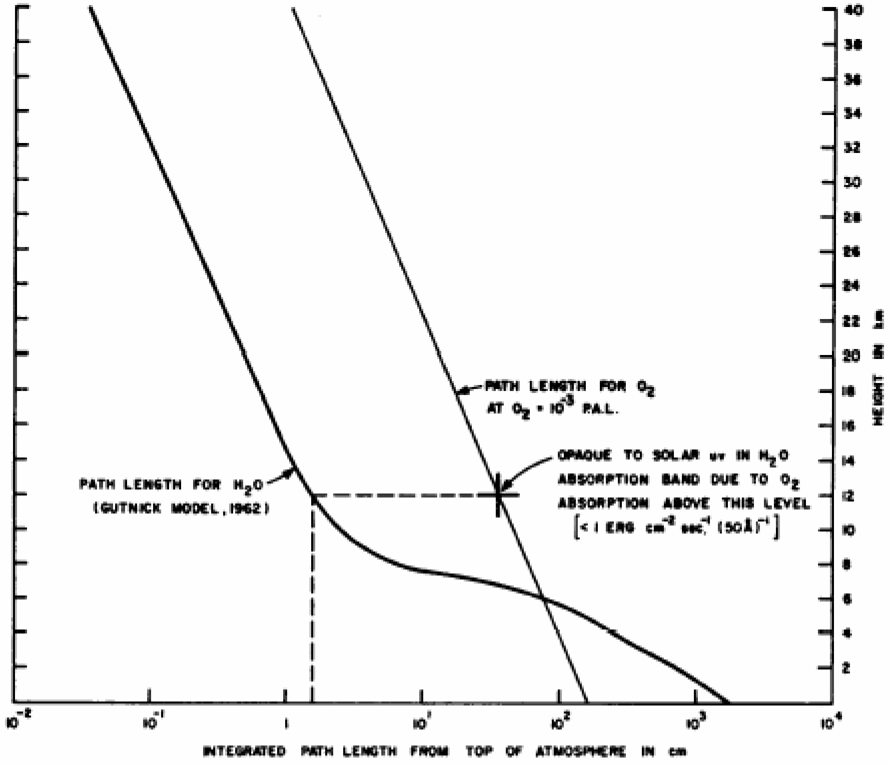


FIG. 6.—Shielding from photodissociation of atmospheric H_2O by O_2.

PNAS 1965;53;1215-1225

- Resulting O_2 mixing ratio is of the order of 10^{-3} to 10^{-4} PAL

  (times the Present Atmospheric Level)

- Don't worry if you can't read this graph, because their conclusion is completely wrong!

Brinkman's model

- Brinkman (*Planet. Space Sci.* **19**, 791-794, 1971) predicted abiotic O_2 concentrations as high as 0.27 PAL

- Sinks for O_2
  - He included a sink due to crustal oxidation, but he neglected volcanic outgassing of reduced species (*e.g*., H_2, CO)
- Source of O_2
  - He assumed that precisely 1/10th of the H atoms produced by H_2O photolysis escaped to space. This fraction is much too high
  - Not until 1973 did we understand what controls the hydrogen escape rate on Earth. Don Hunten (J. Atmos. Sci., 1973) figured this out while studying H escape from Saturn's moon, Titan

Hydrogen escape

- Hydrogen escape can be limited either at the *exobase* (~500 km altitude) or at the *homopause* (~100 km altitude)
- Exobase—the altitude at which the atmosphere becomes collisionless
  - An exobase may not exist in a hydrogen-dominated upper atmosphere ⇒ get hydrodynamic escape
  - In any case, the factor limiting H escape in this case is *energy* (from solar EUV heating)

Mean free path

= local scale height

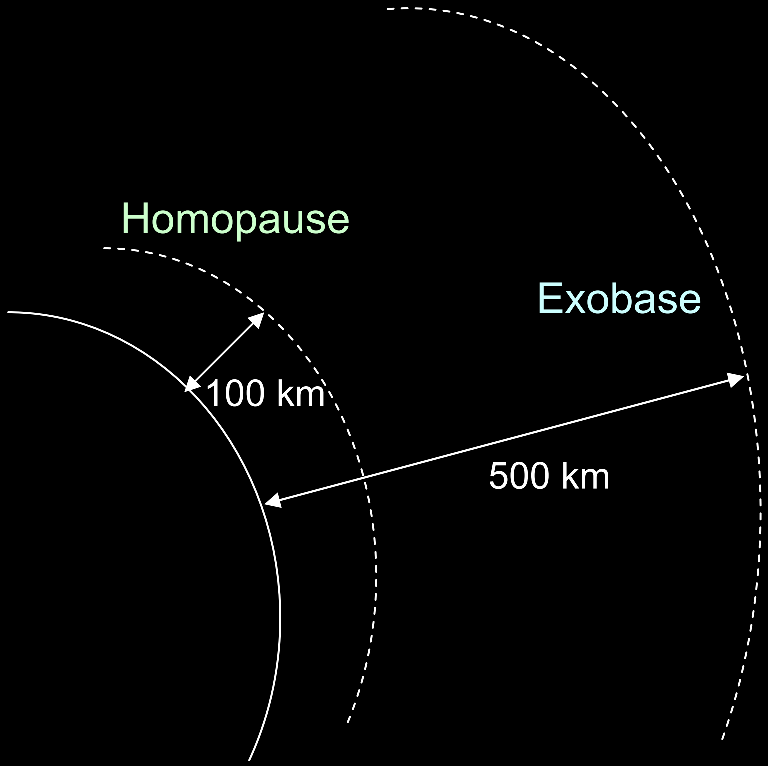
$$\frac{1}{\sigma n} \cong \frac{kT}{mg}$$

σ = molecular collision cross section

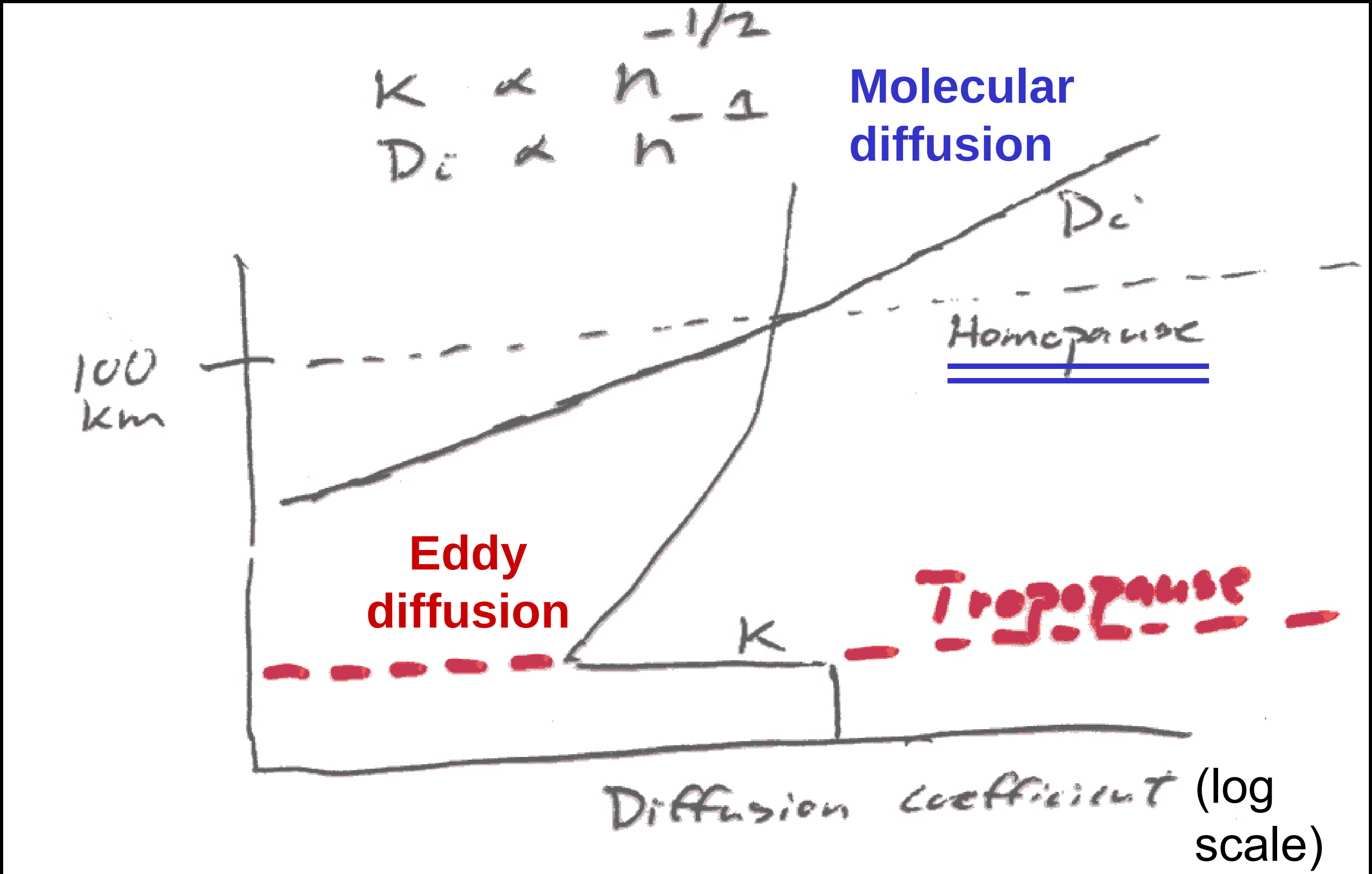
$$
\begin{aligned}
n &\cong \frac{mg}{\sigma kT} \\
&= \frac{16\left(1.67\times10^{-24}\,g\right)\left(850\,cm/s^2\right)}{3\times10^{-15}\,cm^2\,(1.38\times10^{-16}\,erg/K)(1500K)} \\
&\cong 4\times10^{7}\,molecules/cm^3
\end{aligned}
$$

Hydrogen escape (cont.)

- Homopause—the altitude at which molecular diffusion replaces "eddy diffusion" as the dominant vertical transport mechanism
- Light gases separate out from heavier ones above this altitude
- The flux of hydrogen through the homopause is limited by *diffusion*



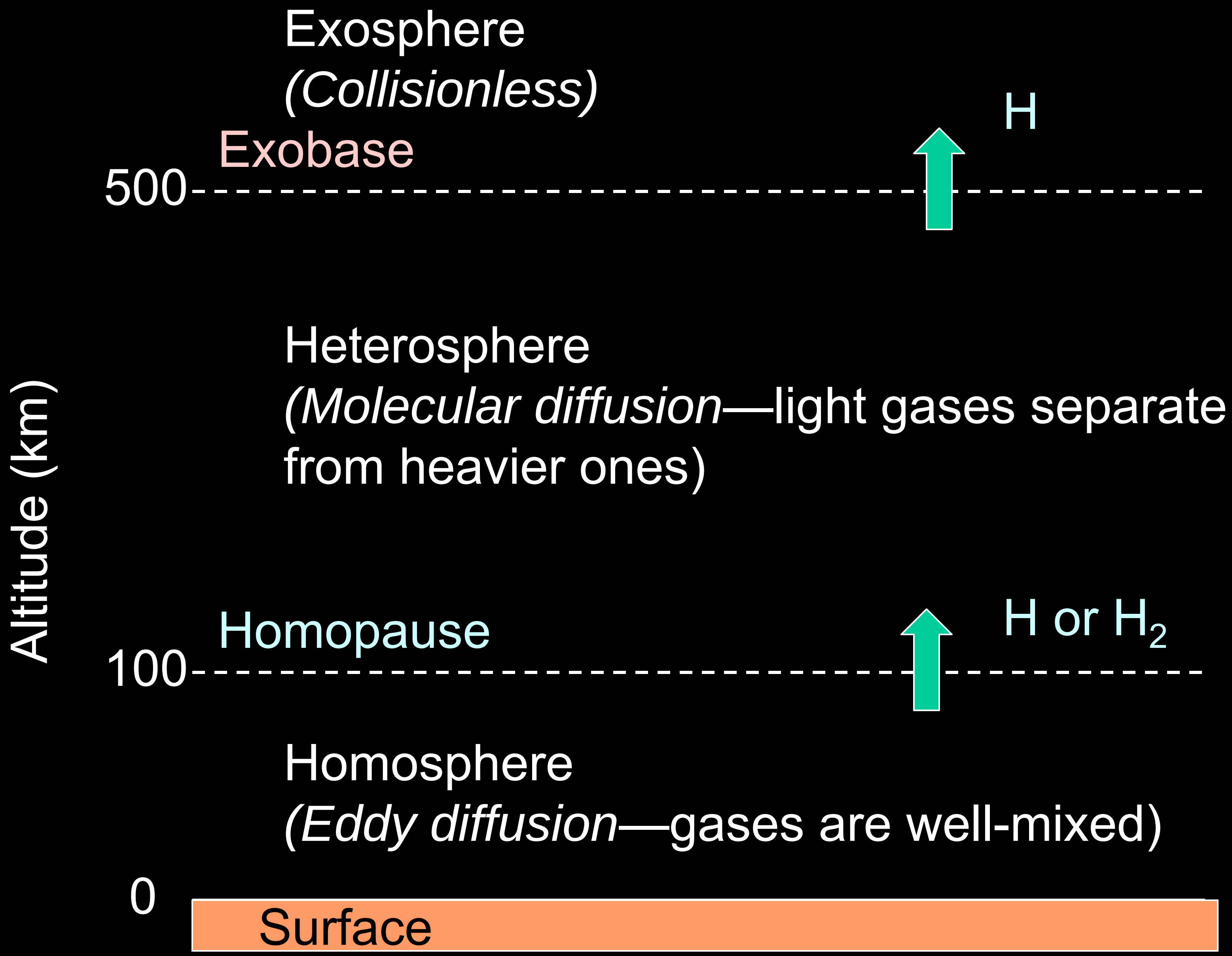
Hydrogen escape (cont.)



Molecular diffusion

Eddy diffusion

(log scale)



Exosphere
(Collisionless)

H

Exobase

500

Heterosphere
(Molecular diffusion—light gases separate from heavier ones)

Altitude (km)

Homopause

H or H_2

100

Homosphere
(Eddy diffusion—gases are well-mixed)

0

Surface

Hydrogen escape from the exobase

- Earth's upper atmosphere is rich in O_2 (a good EUV absorber) and poor in CO_2 (a good IR radiator) ⇒ the exosphere is *hot*

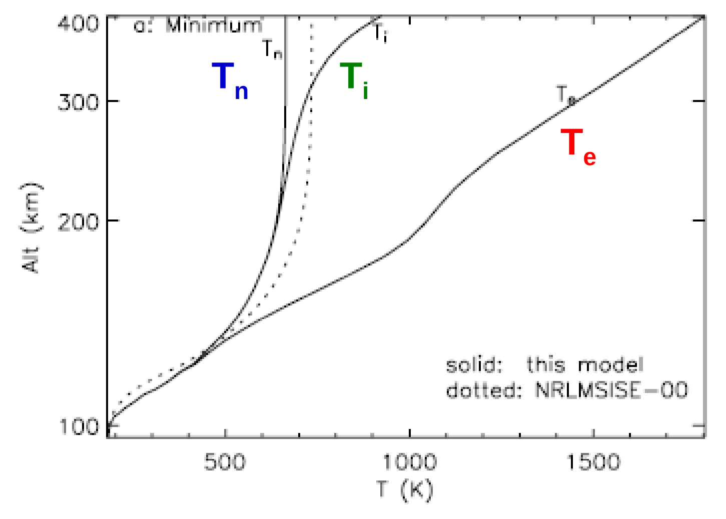
$$T_\infty \cong 700\text{ K (solar min)}$$
$$\cong 1200\text{ K (solar max)}$$

- Furthermore, H_2 is broken apart into H atoms by reaction with hot O atoms

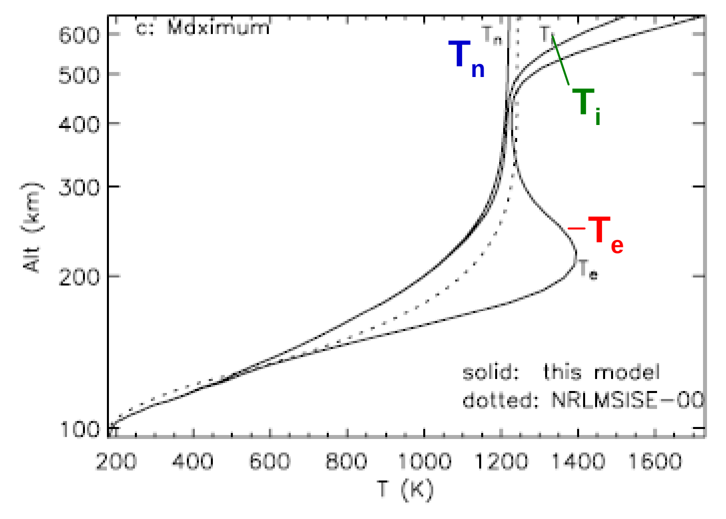
$$H_2 + O \rightarrow H + OH$$
$$OH + O \rightarrow O_2 + H$$

- Escape of light H atoms is therefore relatively easy

Thermospheric temperature profiles for Earth



Solar minimum



Solar maximum

- T_n = neutral temperature
- T_i = ion temperature
- T_e = electron temperature

F. Tian, J.F. Kasting, et al., *JGR* (2008)

Hydrogen escape from the exobase

- For Earth, there are 3 important H escape mechanisms:
  - *Jeans escape*: thermal escape from the high-energy tail of the Maxwellian velocity distribution
  - *Charge exchange* with hot H^+ ions in the magnetosphere
  - The *polar wind*
- Let's consider Jeans escape first $\Rightarrow$

Kinetic theory of gases

- Jeans escape is a form of thermal escape. Jeans' theory relied on previous work by Maxwell
- James Clerk Maxwell (1831-1879)

  > "(The work of Maxwell) ... the most profound and the most fruitful that physics has experienced since the time of Newton."
  >
  > **—Albert Einstein, The Sunday Post**

Image from *Wikipedia*

Maxwellian velocity distribution

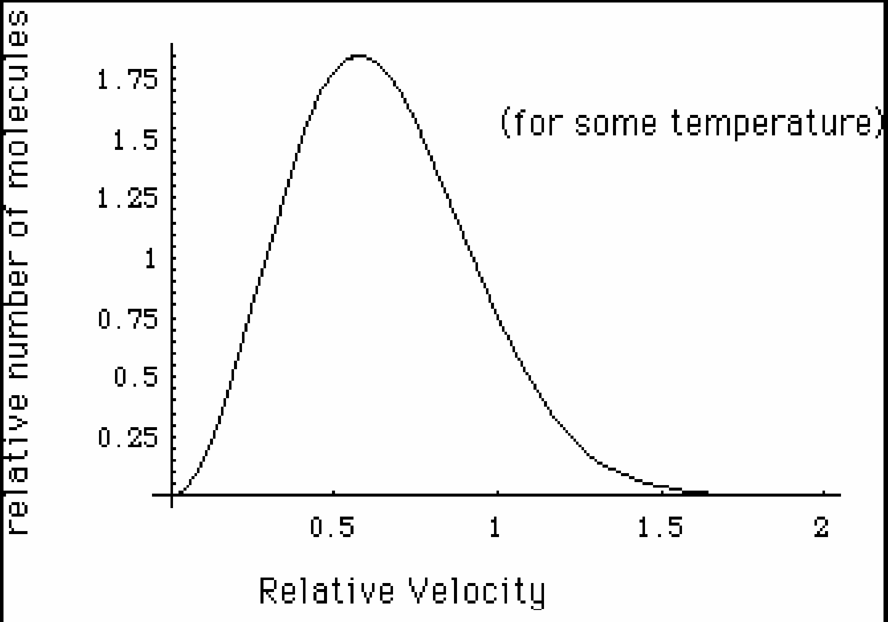
- Let *f(v)* be the number of molecules with speeds between *v* and *v + dv*
- Constants:

k = Boltzmann's constant,

1.38×10^{-23} J/K

m = molecular mass

T = temperature (K)



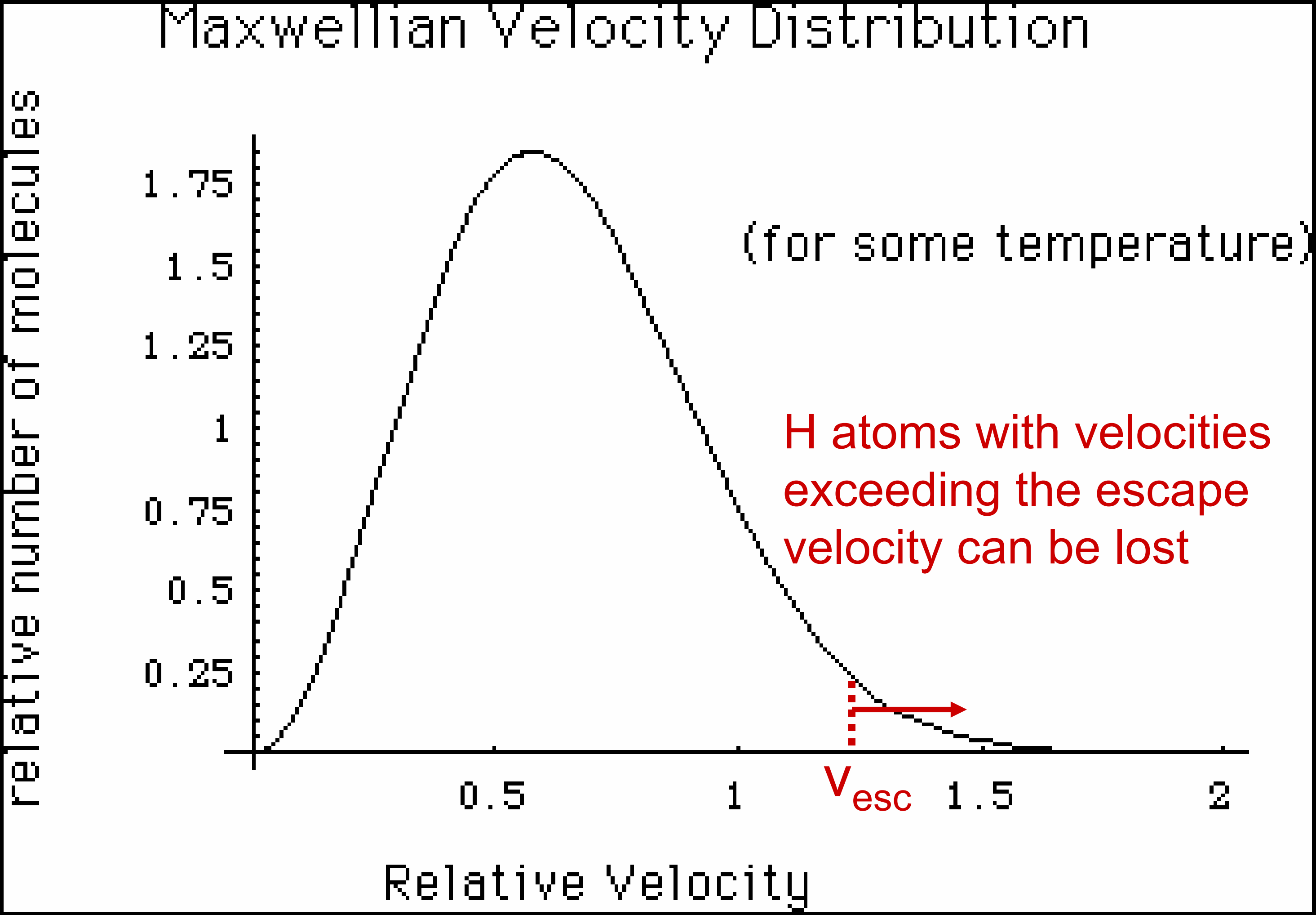
$$f(v)dv = \frac{4n}{\sqrt{\pi}}\left(\frac{m}{2kT}\right)^{3/2} v^2 \exp\left(-\frac{mv^2}{2kT}\right)dv$$

Kinetic theory of gases

- Sir James Jeans (1877-1946)
  - Wrote: *The Dynamical Theory of Gases* (1904)
  - Figured out large chunks of what we now study in physics classes…

Image from *Wikipedia*

Jeans (thermal) escape



H atoms with velocities exceeding the escape velocity can be lost

Escape velocity

- In order to escape, the kinetic energy of an escaping molecule must exceed its gravitational potential energy *and* it must be headed upwards and not suffer any collisions that would slow it down

- Who can do this mathematically?

Escape velocity

$\frac{1}{2} m v_e^2 = GMm/r$

(K.E.) (P.E.)

$v_e = (2\,GM/r)^{1/2}$

$= 10.8$ km/s (at 500 km altitude)

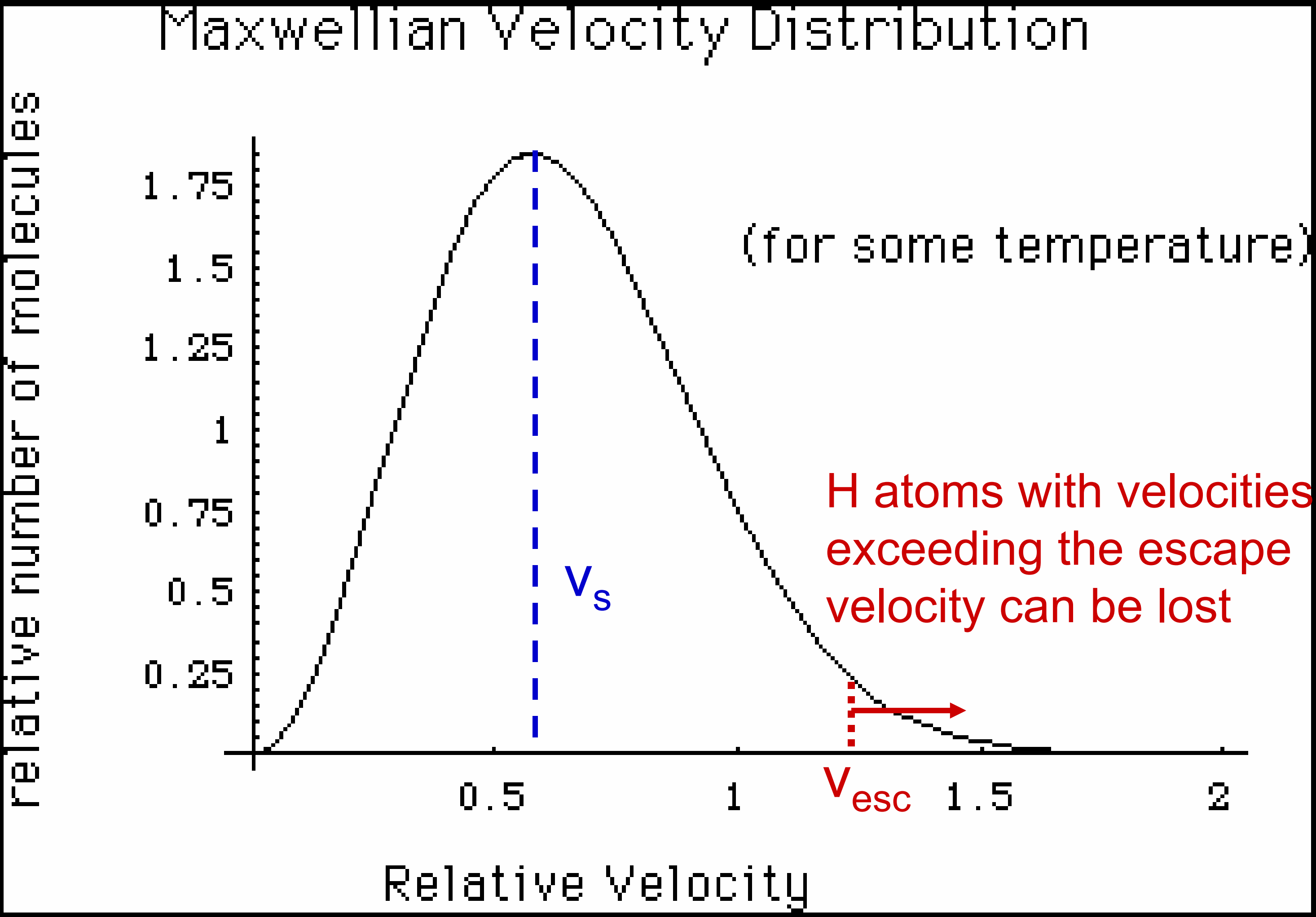
m = mass of atom (1.67×10^{-27} kg for H)

M = mass of the Earth (5.98×10^{24} kg)

G = universal gravitational constant (6.67×10^{-11} N m^2/kg^2)

r = radial distance to the exobase (6.871×10^{6} m)

Most probable velocity



H atoms with velocities exceeding the escape velocity can be lost

Root mean square velocity

Energy: ½ kT per degree of freedom

Translational energy: 3 degrees of freedom

$$\Rightarrow \quad \mathrm{KE} = 3/2\, kT$$

$$\tfrac{1}{2} m v^2 = 3/2\, kT$$

$$v_{rms} = (3\, kT/m)^{1/2}$$

Most probable velocity

- Most probable velocity: $v_s = (2\ kT/m)^{1/2}$
- Evaluate for atomic H at T = 1000 K

  $v_s = 4.07$ km/s
- Compare with escape velocity

  $v_{esc} = 10.8$ km/s
- These numbers are not too different

  $\Rightarrow$ an appreciable number of H atoms can escape

Escape parameter, λ

- Define the escape parameter, λ_c, as the ratio of gravitational potential energy to thermal energy at the critical level, r_c

$$\lambda_c = \frac{GMm/r_c}{\frac{1}{2} m v_s^2} = \frac{GMm/r_c}{\frac{1}{2} m\,(2kT/m)}$$

$$\lambda_c = \frac{GMm}{kTr_c}$$

Jeans' escape flux

The Jean's escape velocity can be calculated by integrating over the Maxwellian velocity distribution, taking into account geometrical effects (escaping atoms must be headed upwards). The result is

$$\phi_{esc} = \frac{1}{2\sqrt{\pi}} n_c v_s (1 + \lambda_c) e^{-\lambda_c}$$

The escape flux is equal to the escape velocity times the number density of hydrogen atoms at the critical level, or exobase

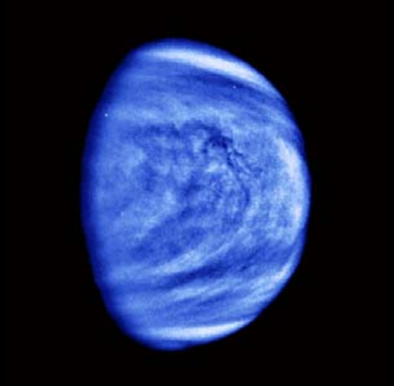
$$\phi_{esc} = n_c v_J$$

- If the exospheric temperature is high, then Jeans’ escape is efficient and hydrogen is easily lost
  - In this case, the rate of hydrogen escape is determined at the homopause (diffusion-limited flux)
- If the exospheric temperature is low, then hydrogen escape *may* be bottled up at the exobase

Hydrogen escape processes

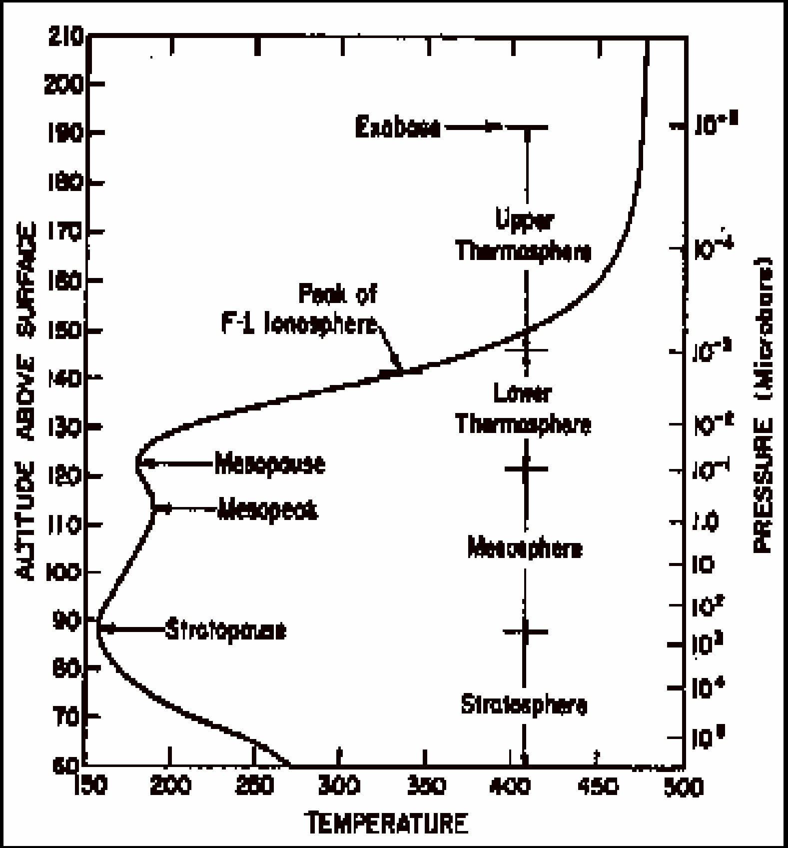
- Mars and Venus have CO_2-dominated upper atmospheres which are very cold (350-400 K)

  ⇒ Escape from the exobase is limiting on both planets



Venus dayside temperature profile

- Upper atmosphere is relatively cool, despite being strongly heated by the Sun
- CO_2 is a good infrared *radiator*, as well as absorber



http://www.atm.ox.ac.uk/user/fwt/WebPage/Venus%20Review%204.htm

Hydrogen escape processes

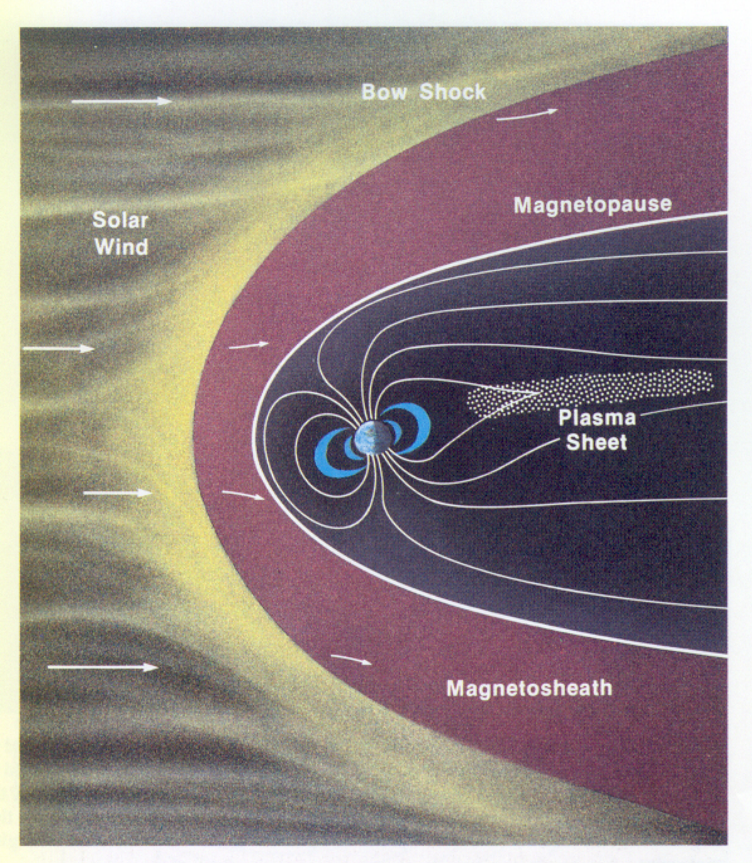
- For Earth, Jeans escape is efficient at solar maximum but not at solar minimum
  - However, there are also other *nonthermal* H escape processes that can operate..

Nonthermal escape processes

- Charge exchange with hot H^+ ions from the magnetosphere

$H + H^+$ (hot)

$\rightarrow$ $H^+ + H$ (hot)

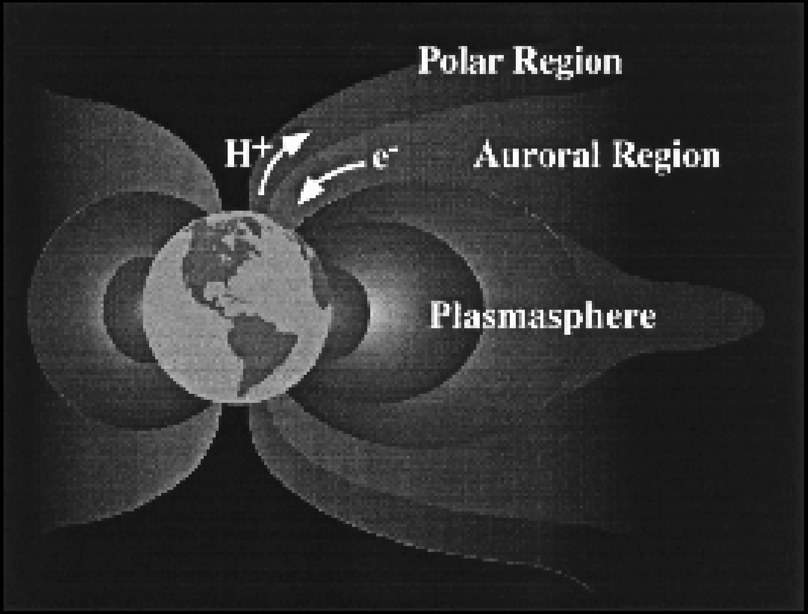


The New Solar System, ed., 3, p. 35

Nonthermal escape processes

- The polar wind:

  H^+ ions can be accelerated out through open magnetic field lines near each pole
- The upward acceleration is set up by a charge separation electric field that exists in the ionosphere
  - Electrons are lighter than the dominant O^+ ions; hence, they tend to diffuse to higher altitudes



http://www.sprl.umich.edu/SPRL/research/polar_wind.html

Conclusion: Hydrogen can escape efficiently from the present exobase at both solar maximum and solar minimum

⇒ H escape is limited by diffusion through the homopause

Corollary: The escape rate is easy to calculate (see next lecture)