Preliminary lecture programme:

1. The particle universe: introduction, cosmological parameters
2. Basic cross sections for neutrinos and gamma-rays; IceCube
3. Density of relic particles from the early Universe
4. Dark matter: Direct and indirect detection methods; the galactic centre & other promising DM sources
5. Neutrinos and antimatter from dark matter, Sommerfeld enhancement
6. Particular dark matter candidates (WIMPS, Kaluza-Klein particles, sterile neutrinos,…)
7. Supersymmetric dark matter, DarkSUSY
8. Diffuse extragalactic gamma-rays, Primordial black holes, Hawking radiation
9. Gravitational waves

Einstein's equation for general relativity:

$$G^{\mu\nu} = 8\pi G_N T^{\mu\nu}$$

curvature

energy-momentum

factor $8\pi G_N$ given by Newtonian limit

$$G^{\mu\nu} = R^{\mu\nu} - \frac{1}{2} g^{\mu\nu} R$$

Ricci curvature tensor

Ricci scalar

$R^{\mu\nu}$ is formed from the metric tensor $g^{\mu\nu}$ and its derivatives

Special relativity	$\rightarrow$	general relativity
$\eta^{\mu\nu}$	$\rightarrow$	$g^{\mu\nu}$
∂^{μ}	$\rightarrow$	D^{μ}
Calculations by hand	$\rightarrow$	Matlab, Mathematica, Maple,…

Weak gravitational fields $\Rightarrow$ $$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} \sim 1 + \mathcal{O}(10^{-20})$$

Series expansion should be excellent.

We first recall the way we derive the existence of electromagnetic waves in Maxwell's theory. We insert the vector potential A_μ in the equations of motion for a vanishing current j_μ (that is, in vacuum) to obtain

$$\Box A^\mu - \partial^\mu \left(\partial_\nu A^\nu\right) = 0$$

where the d'Alembertian wave operator is

$$\Box \equiv \partial^\mu \partial_\mu = \frac{\partial^2}{\partial t^2} - \nabla^2$$

Through the use of the gauge freedom,

$$A^\mu \rightarrow A^\mu + \partial^\mu f$$

we can choose A^μ to fulfill the axial condition $A^0 = 0$ (free-field) and the Lorentz condition $$\partial_\nu A^\nu = 0$$

This immediately leads to the simple wave equation

$$\Box A^\mu = 0$$

which has solutions of the form

$$A^\mu(\mathbf{r},t) = \epsilon^\mu e^{\pm i(\omega t - \mathbf{k}\cdot\mathbf{r})} = \epsilon^\mu e^{\pm i k^\nu x_\nu}$$

with

$$k^\mu k_\mu = 0$$

(light-like propagation) and the gauge conditions

$$A^0 = 0;\ \partial_\nu A^\nu = \nabla \cdot \mathbf{A} = 0$$

(A^0 = 0 since there are no sources, "radiation gauge"). These translate into

$$\epsilon^0 = 0 \text{ and } \mathbf{k}\cdot\epsilon = 0,$$

showing that the two physical
degrees of freedom are transverse to the direction of propagation.

By superposition of, for example, a wave linearly polarized in the x-direction and one in the y-direction phase shifted by 90 degrees (represented by multiplication of the amplitude by the imaginary unit i), we obtain circularly polarized states, corresponding to definite photon helicity. Thus, two of the components of the 4-vector A^μ have disappeared, and only two physical polarizations remain. (This only works for massless fields like the photon. In general a spin-1 particle should have 3 polarizations.)The polarization 4-vectors

$$\epsilon^\mu_\pm = (0, 1, \mp i, 0)/\sqrt{2}$$

carry the properties (including the gauge choice) of the electromagnetic radiation.

We now try to do a similar construction for gravity waves. Now we have a tensor field

$$h_{\mu\nu}$$

instead of the vector field A_μ, but again we can use a gauge-like invariance (or rather reparametrization invariance)

$$x_\mu \to x_\mu + \xi_\mu(x)$$

This translates into

$$h_{\mu\nu} \to h_{\mu\nu} - \partial_\mu \xi_\nu - \partial_\nu \xi_\mu$$

and we can use this freedom to choose the gauge condition

$$h^{\mu}{}_{\mu} = 0$$

(traceless gauge). The analogy of $A_0 = 0$ is

$$h_{0\nu} = h_{\nu 0} = 0,$$

and of the transversality condition $\nabla \cdot \mathbf{A} = 0$ it is

$$\nabla_i h^{i\nu} = \nabla_i h^{\nu i} = 0$$

The Einstein equation (negecting back-reaction, i.e. the contribution to the energy-momentum tensor by the gravitional field itself) becomes simply

$$\Box h_{\mu\nu} = 0$$

Exactly like for photons we can write for the wave solutions to Einsteins equation:

$$h_{\mu\nu} = E_{\mu\nu} e^{\pm i(\omega t - \mathbf{k}\cdot\mathbf{x})}$$

with $\omega^2 = k^2$, i.e. massless propagation (with the speed of light).

We can represent $E_{\mu\nu}$ by a 4x4 matrix, which, exactly like for A_μ, should reflect the gauge choice. We know already that the $E_{0\nu}$ row and $E_{\mu 0}$ columns are zero. Also E has to be symmetric in the two indices (since the metric is). Further, $k^i E_{i\nu} = k^j E_{\mu j} = 0$, meaning that also the elements of the $E_{3\nu}$ column and $E_{\mu 3}$ row are zero for a wave propagating in the z-direction. So, we really just have zeros apart from a symmetric, traceless 2x2 matrix. A general such matrix is a linear combination of

$$E^{+}_{\mu\nu} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{and} \quad E^{\times}_{\mu\nu} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

For a given value of the 3-component z, and at time t, we can then write

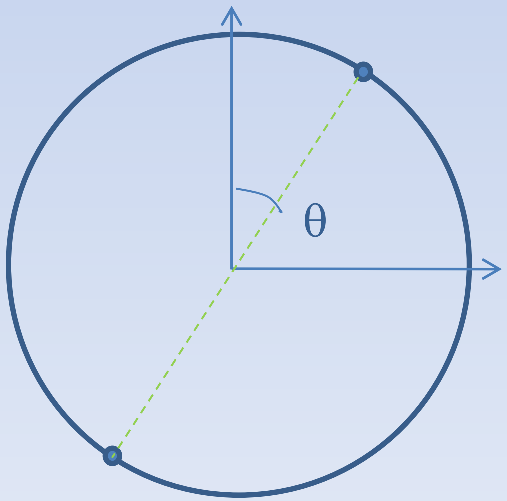
$$E_{\mu\nu}(t) = h_+(t)E^+_{\mu\nu} + h_\times(t)E^\times_{\mu\nu}.$$

Look at the case

$$h_+ \neq 0, \quad h_\times = 0.$$

At a given time, we have in the unperturbed case

$$\Delta s^2 = \eta_{\mu\nu}\Delta x^\mu \Delta x^\nu = (\Delta t)^2 - \sum_i (\Delta x^i)^2 = -\sum_i (\Delta x^i)^2$$



For two diametrically opposed points on the unit circle,

$$\Delta x^i = (2\cos\theta, 2\sin\theta, 0)$$

and their distance is

$$d_0 = \sqrt{-\eta_{ij}(\Delta x^i)(\Delta x^j)} = 2\sqrt{\sin^2\theta + \cos\theta^2} = 2$$

In the perturbed case (i.e., if a gravity wave passes)

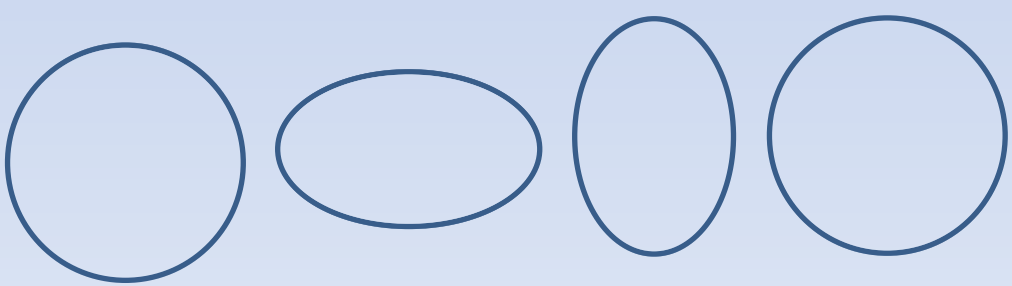
$$d_+ = \sqrt{-(\eta_{ij} + h_+ E^+_{ij})\Delta x^i \Delta x^j} = \sqrt{4 - h_+(t)4(\cos^2\theta - \sin^2\theta)} \simeq$$

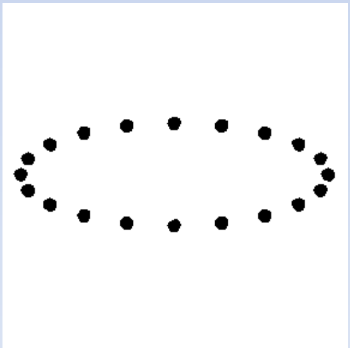
$$2 - h_+(t)(cos^2\theta - \sin^2\theta) = 2 - h_+(t)\cos 2\theta$$

Foor simplicity, we may work with real h by combining as usual the waves with the two signs in the exponential, giving

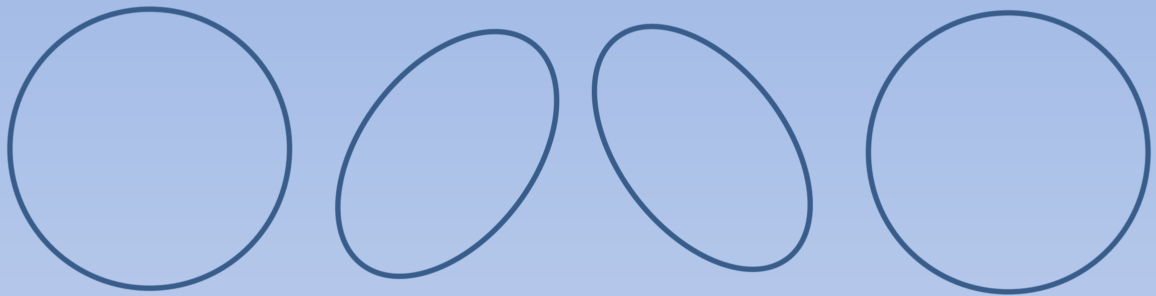
$$h^+_{\mu\nu} = E^+_{\mu\nu} h_+(t) = E^+_{\mu\nu}\cos\left(\omega t - \mathbf{k}\cdot\mathbf{r}\right)$$

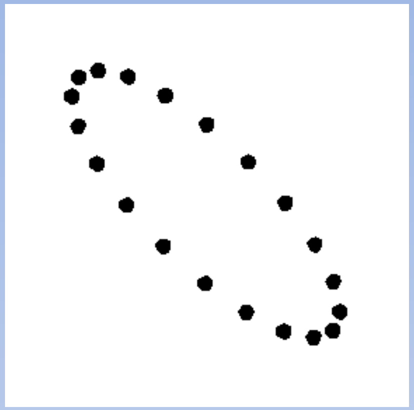
and we see that the unit circle will be successively "compressed" or "squeezed" depeding on the sign of the last factor





A similar analysis for the $\times$ case gives (exercise: show this!)





These are the two independent quadrupole deformations of a circle. This means that the source of the gravitational field giving gravity waves has to have a quadrupole moment.

From dimensional reasoning, $\boxed{h \sim \frac{G_N \ddot{Q}}{d} \sim \frac{4 G_N E_{\rm kin}}{d}}$,

since

$$Q \sim M l^2 \implies \dot{Q} = M 2 l \dot{l} = 2 M l v \implies \boxed{\ddot{Q} \sim 2 M v^2 = 4 E_{kin}}$$

Typical example: coalescing binary stars.

$E_{\rm kin} \sim M_\odot \implies h \sim 10^{-17}$ for d ~ 10 kpc (Milky Way)

$E_{\rm kin} \sim M_\odot \implies h \sim 10^{-20}$ for d ~ 15 Mpc (VIRGO)

Gravity waves have already been indirectly detected:

Hulse & Taylor (Nobel Prize 1993) binary pulsar PSR 1913-16

Measured decrease in orbital period time:

$$\frac{dP}{dt} = (-2.4225 \pm 0.0056) \cdot 10^{-12}$$

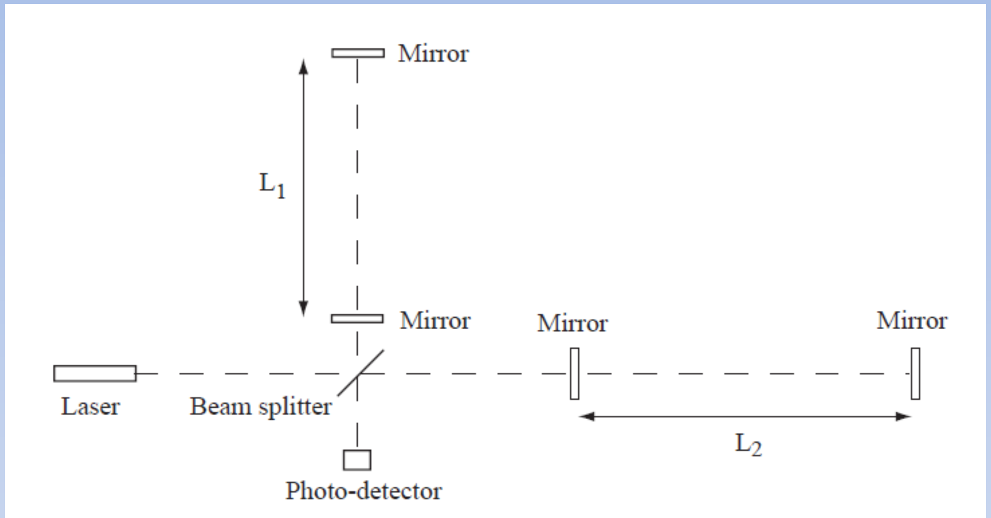
General relativistic calculation (energy loss due to gravitational radiation):

$$\frac{dP_{GR}}{dt} = -2.40 \cdot 10^{-12}$$

This excellent agreement has put severe limits on modifications of Einstein gravity.

- But effects of gravity waves have so far never been detected directly on Earth…

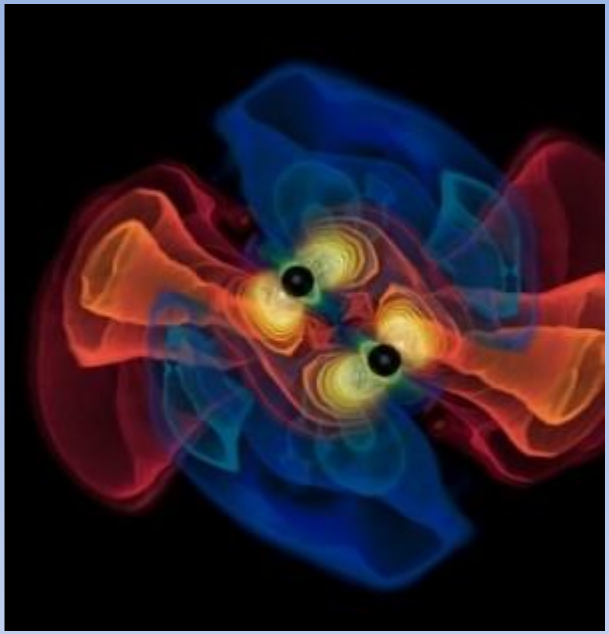
LIGO East:



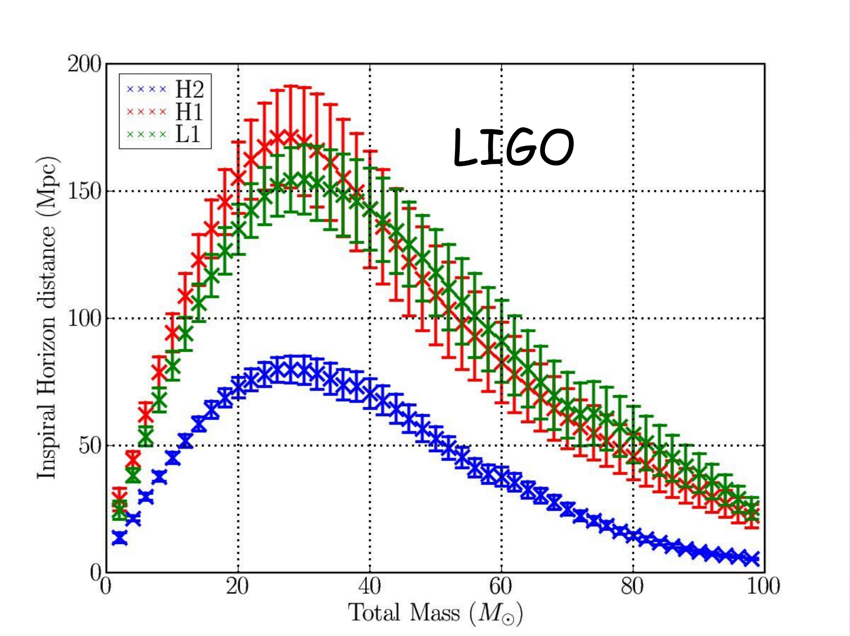
Laser interferometer, principle
(Michelson - Morley, Fabry-Perot)

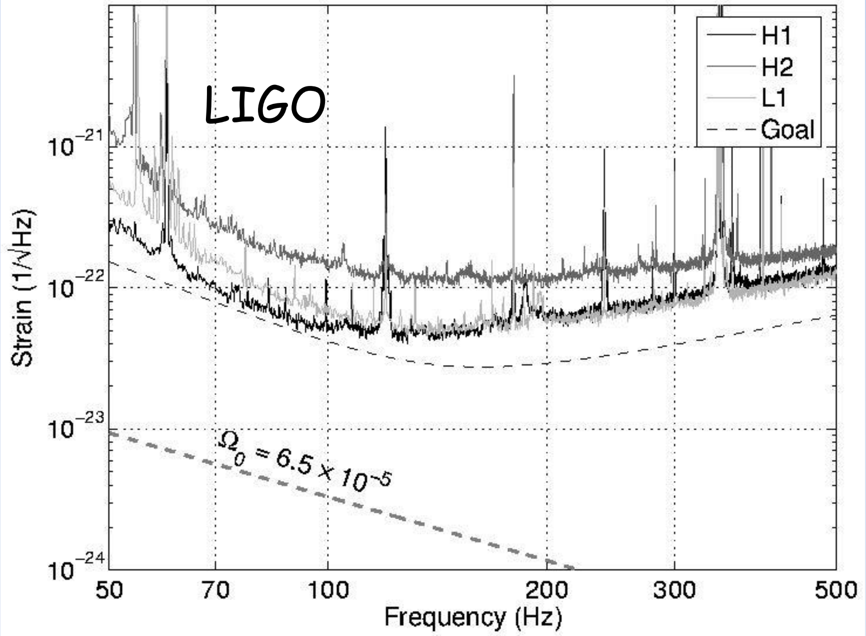
LIGO site, Livingston, Louisiana
(also a LIGO site in Hanford)

Also VIRGO detector in Italy and GEO600 in Germany. Coincidence runs are now made with all these detectors in a global gravity wave network.

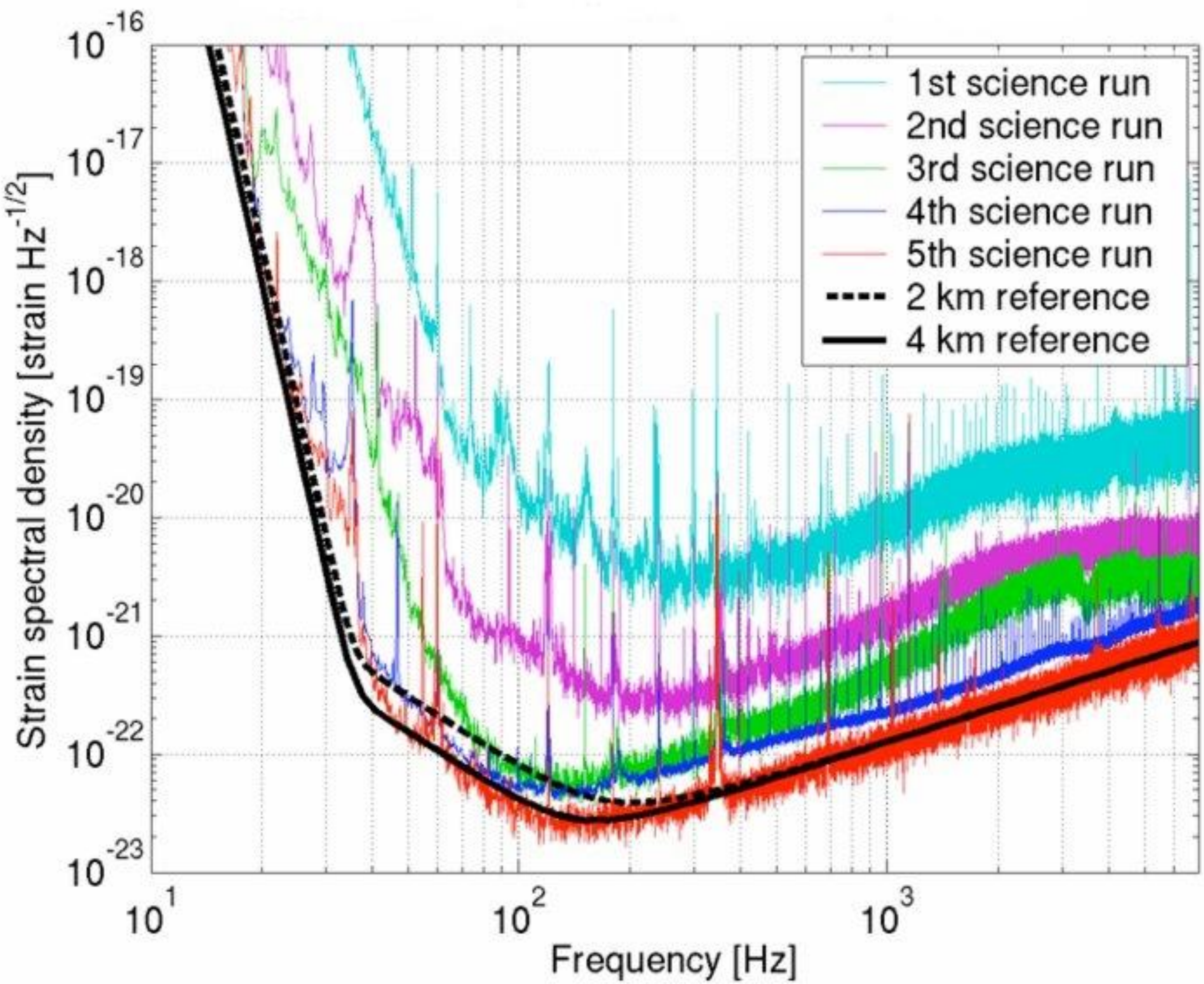


Coalescing black holes (NIKHEF)





LIGO sensitivity

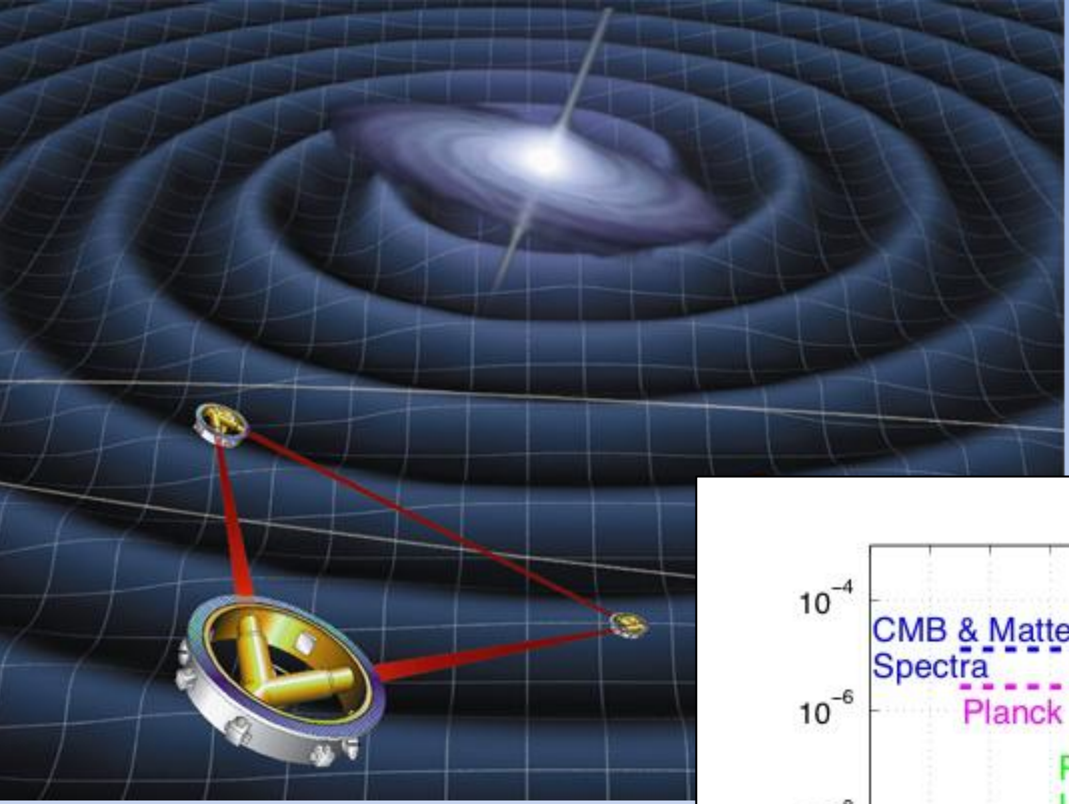


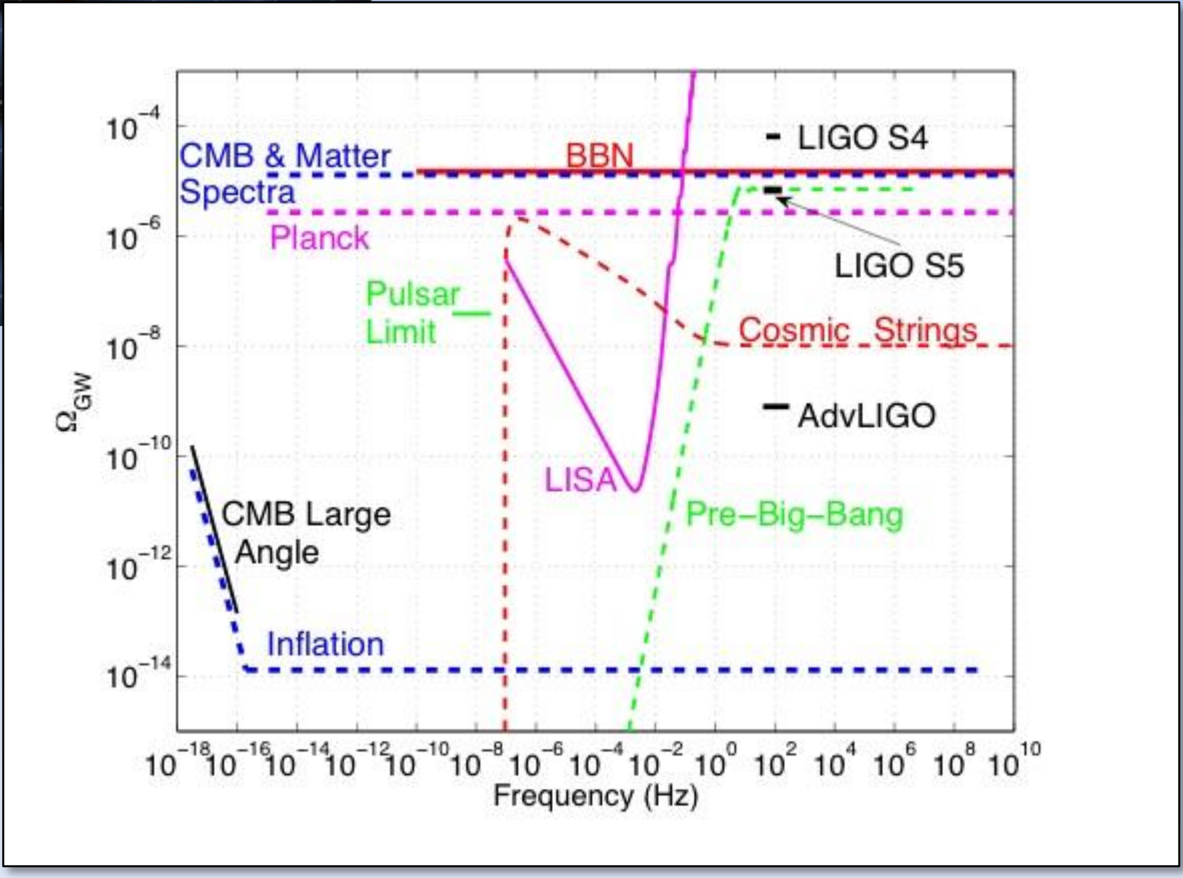
Unfortunately, no signal detected yet…

So LIGO is in the same situation as, e.g., IceCube: No sources yet, but we are all waiting to pop open the Champagne bottles…

The space mission LISA will have 5 million km armlength…

LISA





LIGO S4
CMB & Matter Spectra
BBN
Planck
LIGO S5
Pulsar Limit
Cosmic Strings
AdvLIGO
LISA
Pre–Big–Bang
CMB Large Angle
Inflation
Ω_{GW}
10^{-4} 10^{-6} 10^{-8} 10^{-10} 10^{-12} 10^{-14}
10^{-18} 10^{-16} 10^{-14} 10^{-12} 10^{-10} 10^{-8} 10^{-6} 10^{-4} 10^{-2} 10^{0} 10^{2} 10^{4} 10^{6} 10^{8} 10^{10}
Frequency (Hz)

All-sky search for gravitational-wave bursts in the first joint LIGO-GEO-Virgo run

"No events produced by the search algorithms survive the selection cuts."

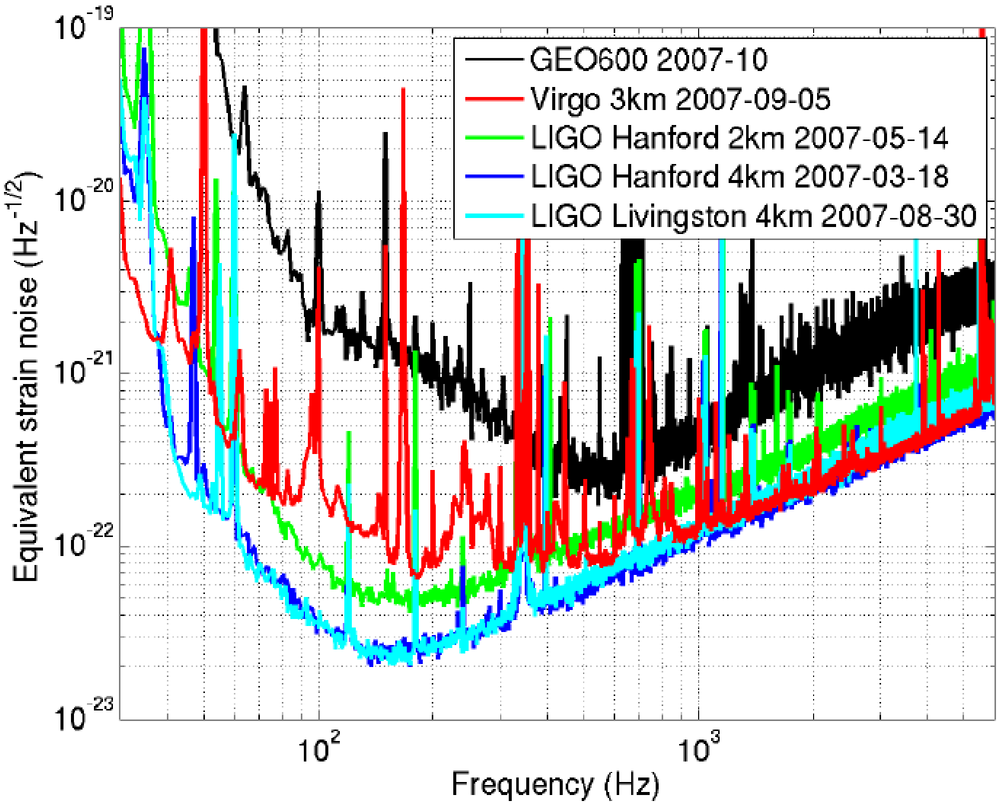


FIG. 1: Best noise amplitude spectral densities of the five LSC/Virgo detectors during S5/VSR1.

J. Abadie & al., arXiv:1002.1036